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Hexagonal and trigonal sphere packings. IV. Trivariant lattice complexes of trigonal space groups

Heidrun Sowa*‡ and Elke Koch

Institut für Mineralogie, Petrologie und Kristallographie der Philipps-Universität Marburg, Hans-Meerwein-Strasse, D-35032 Marburg, Germany. Correspondence e-mail: heidrun.sowa@geo.uni-goettingen.de

The 13 trivariant lattice complexes with trigonal symmetry are compatible with 218 types of homogeneous sphere packings, 7 types of interpenetrating sphere packings and one type of interpenetrating layers of spheres. Altogether, the lattice complexes with trigonal characteristic space group (with 0, 1, 2 or 3 degrees of freedom) give rise to 225 types of sphere packing. 110 of these types are compatible exclusively with one of the 13 trivariant lattice complexes, 31 in addition with some of the invariant, univariant or bivariant lattice complexes, whereas 6 types occur solely in such a lattice complex. 65 types encompass special sphere packings that can also be generated with hexagonal symmetry [Sowa, Koch & Fischer (2003). *Acta Cryst.* A**59**, 317–326; Sowa & Koch (2004). *Acta Cryst.* A**60**, 158–166; Sowa & Koch (2005). *Acta Cryst.* A**61**, 331–342]; cubic inherent symmetry occurs for certain sphere packings [Fischer (2004). *Acta Cryst.* A**60**, 246–249] belonging to 13 types. The maximal inherent symmetry is trigonal for 147 of the 225 types. The sphere packings of 61 types can be subdivided into connected layer-like subunits, those of 86 types into connected rod-like subunits. Such subunits may be used to construct some kind of 'descriptive symbols' that reflect certain properties of the sphere packings. Interpenetrating sphere packings with cubic inherent symmetry belong to one of the 7 types. All interpenetrating sphere layers that belong to the only type occurring in the trigonal crystal system show hexagonal inherent symmetry. Some examples depict crystal structures that can be described by means of sphere packings.

1. Introduction

In three previous papers, tables have been presented that list all types of homogeneous sphere packings, interpenetrating sphere packings and interpenetrating sphere layers that can be generated in lattice complexes with trigonal or hexagonal characteristic symmetry with less than three degrees of freedom (Sowa *et al.*, 2003; Sowa & Koch, 2004) or in the 15 trivariant lattice complexes with hexagonal characteristic symmetry (Sowa & Koch, 2005).

It is the aim of the present paper to complete this material by information on the sphere packings, interpenetrating sphere packings and interpenetrating layers of spheres that correspond to point configurations of the 13 trigonal trivariant lattice complexes. Only some of the sphere packings that refer to the general positions of $R\bar{3}$, $R\bar{3}c$ (Sowa & Koch, 1999) or $P3_221$ (Sowa, 2003) have been tabulated before.

All necessary definitions and the information on the derivation of the sphere packings may be taken from the preceding papers.

2. Results

For the 13 trigonal trivariant lattice complexes, significant data on all types of sphere packings, interpenetrating sphere packings and layers are summarized in Table 1. The information is distributed among three blocks.

(i) Each lattice complex is designated by its characteristic Wyckoff position. In order to derive all sphere packings, only one asymmetric unit of the Euclidean normalizer of the characteristic space group has to be investigated (*cf. e.g.* Koch *et al.*, 2002). The corresponding range of the coordinate parameters is given in addition.

(ii) In the second block, capital letters are assigned to the coordinate triplets of all possible neighbouring points. If, for symmetry reasons, two or more neighbouring points are equidistant – irrespective of the choice of the free coordinate

‡ Present address: GZG Abt. Kristallographie, Georg-August-Universität Göttingen, Goldschmidtstrasse 1, D-37077 Göttingen, Germany.

Table 1
Sphere packings, interpenetrating sphere packings and interpenetrating sphere layers corresponding to the 13 trivariant trigonal lattice complexes.

$P\bar{3}$ 6*g*			**0, 2*x*−1 ≤ *y* ≤ ½*x*; 0 ≤ *z* ≤ ¼**						
A	*x*−*y*, *x*, −*z*		*C*	−*x*+1, −*y*, −*z*	*E*	−*y*, *x*−*y*, *z*	*G*	−*x*+1, −*y*, −*z*+1	
	y, −*x*+*y*, −*z*					−*x*+*y*, −*x*, *z*			
B	*x*, *y*, *z*+1		*D*	−*y*+1, *x*−*y*, *z*	*F*	*x*−*y*, *x*, −*z*+1	*H*	−*y*, *x*−*y*−1, *z*	
	x, *y*, *z*−1			−*x*+*y*+1, −*x*+1, *z*		*y*, −*x*+*y*, −*z*+1		−*x*+*y*+1, −*x*, *z*	
0.1	10/3/*h*5	*ABCDFG*	$\frac{46}{73}-\frac{20}{219}\sqrt{6}$, $\frac{14}{219}\sqrt{6}-\frac{3}{73}$, ¼, $2(\frac{13}{73}-\frac{4}{73}\sqrt{6})^{1/2}$				0.63648		
0.2	12/3/*h*1	*ACDEFGH*	⅓, 0, ¼, $\frac{2}{3}\sqrt{2}$				0.74048		
1.1	6/3/*h*21	*BCDG*	$\frac{4}{7}$, $\frac{1}{7}$, ¼, $\frac{2}{7}$				0.29613	$\frac{2}{7} \le c < 2(\frac{13}{73}-\frac{4}{73}\sqrt{6})^{1/2}$	
1.2	7/3/*h*13	*ABCD*	$\frac{3}{7}$, $\frac{1}{7}$, 0; $\frac{1}{7}\sqrt{7}$				0.51823	$\frac{1}{7}\sqrt{7} \le c < 2(\frac{13}{73}-\frac{4}{73}\sqrt{6})^{1/2}$	
1.3	8/3/*h*3	*ABCFG*	⅓, 0, ¼; $\frac{2}{9}\sqrt{3}$				0.53742	$\frac{2}{9}\sqrt{3} \le c < 2(\frac{13}{73}-\frac{4}{73}\sqrt{6})^{1/2}$	
1.4	8/3/*h*11	*ABDF*	$\frac{2}{5}$, ⅕, ¼, $\frac{2}{5}$				0.58042	$\frac{2}{5} \le c < 2(\frac{13}{73}-\frac{4}{73}\sqrt{6})^{1/2}$	
1.5	8/3/*h*14	*ACDFG*	0.38762, 0.09006, ¼; 0.57982				0.59135	$2(\frac{13}{73}-\frac{4}{73}\sqrt{6})^{1/2} < c < \frac{2}{3}\sqrt{2}$	
1.6	8/3/*h*10	*ADEF*	⅓, ⅙, ¼, ⅓$\sqrt{6}$				0.55536	$\frac{1}{3}\sqrt{6} \le c < \frac{2}{3}\sqrt{2}$	
1.7	6/3/*h*13	*CDGH*	½, 0, ¼, 1				0.45345	$\frac{2}{3}\sqrt{2} < c \le 1$	
2.1	5/3/*h*5	*BCD*	⅓$\sqrt{3}$, ⅔$\sqrt{3}$−1, 0; 2−$\sqrt{3}$				0.26045	$2-\sqrt{3} \le c < 2(\frac{13}{73}-\frac{4}{73}\sqrt{6})^{1/2}$	
2.2	4/3/*h*3	*CDG*	⅙$\sqrt{19}-\frac{1}{6}$, ⅓$\sqrt{19}-\frac{4}{3}$, ¼, ⅓$(6\sqrt{19}-24)^{1/2}$				0.24427	$\frac{2}{7} < c < 1$	
2.3	5/4/*h*5	*ABC*	⅓, 0, 0; ⅓				0.40307	$\frac{1}{3} \le c < 2(\frac{13}{73}-\frac{4}{73}\sqrt{6})^{1/2}$	
2.4	6/3/*h*20	*ABD*	$1-\frac{1}{3}\sqrt{3}$, ½−⅙$\sqrt{3}$, 0; ½$\sqrt{3}-\frac{1}{2}$				0.48601	$\frac{1}{2}\sqrt{3}-\frac{1}{2} \le c < 2(\frac{13}{73}-\frac{4}{73}\sqrt{6})^{1/2}$	
2.5	6/4/*h*2	*ACFG*	⅓, 0, ¼; ⅓$\sqrt{2}$				0.52360	$\frac{2}{9}\sqrt{3} < c < \frac{2}{3}\sqrt{2}$	
2.6	6/3/*h*22	*ADF*	$\frac{13}{12}-\frac{1}{12}\sqrt{73}$, $\frac{13}{24}-\frac{1}{24}\sqrt{73}$, ¼, ⅙$(39-3\sqrt{73})^{1/2}$				0.51755	$\frac{2}{5} < c < \frac{2}{3}\sqrt{2}$	
$R\bar{3}$ 18*f*			**0 < *x* ≤ ⅓; 0 ≤ *y* ≤ ½*x*; 0 ≤ *z* < ½**						
A	*x*−*y*, *x*, −*z*		*D*	−*x*+⅔, −*y*+⅓, −*z*+⅓	*G*	−*x*+⅔, −*y*+⅓, −*z*+⅔	*K*	−*x*+1, −*y*, −*z*	
	y, −*x*+*y*, −*z*				*H*	−*x*+⅓, −*y*−⅓, −*z*+⅔	*L*	−*y*+1, *x*−*y*, *z*	
B	*x*, *y*, *z*+1		*E*	−*y*, *x*−*y*, *z*	*I*	*x*−*y*+⅓, *x*−⅓, −*z*+⅔		−*x*+*y*+1, −*x*+1, *z*	
	x, *y*, *z*−1			−*x*+*y*, −*x*, *z*		*y*+⅓, −*x*+*y*+⅔, −*z*+⅔	*M*	−*x*+1, −*y*, −*z*+1	
C	−*y*+⅓, *x*−*y*−⅓, *z*−⅓		*F*	*x*−*y*, *x*, −*z*+1	*J*	*x*−*y*−⅓, *x*−⅔, −*z*+⅓	*N*	−*y*, *x*−*y*−1, *z*	
	−*x*+*y*+⅔, −*x*+⅓, *z*+⅓			*y*, −*x*+*y*, −*z*+1		*y*+⅔, −*x*+*y*+⅓, −*z*+⅓		−*x*+*y*+1, −*x*, *z*	
0.1	8/3/*h*1	*BCDFG*	$\frac{1}{30387}(12805-1356\sqrt{15})$, $\frac{40}{30387}(137-24\sqrt{15})$, 5/12; $\frac{2}{10129}[30387(137-24\sqrt{15})]^{1/2}$				0.56792		
0.2	9/3/*h*1	*ABCDF*	$\frac{1}{4129}(2063-292\sqrt{15})$, $\frac{8}{4129}(113-24\sqrt{15})$, ¼; $\frac{2}{4129}[12387(113-24\sqrt{15})]^{1/2}$				0.63410		
0.3	9/3/*h*2	*ACDEF*	$\frac{17}{91}$, $\frac{1}{91}$, ¼, $\frac{2}{91}\sqrt{546}$				0.65911		
0.4	8/3/*h*2	*CDFGI*	⅓, $\frac{1}{14}\sqrt{57}-\frac{17}{42}$, 5/12, ⅐$(54\sqrt{57}-390)^{1/2}$				0.52528		
0.5	10/3/*h*1	*CDEFHI*	$\frac{5}{21}$, $\frac{1}{21}$, ⅓; ⅐$\sqrt{42}$				0.63470		
0.6	8/3/*h*3	*ACDJK*	⅓, 0, 1/12; ⅔$\sqrt{3}$				0.53742		
0.6′		*CDHIJ*	⅓, 0, ¼;						
(0.6)		*CFHIM*	⅓, 0, 5/12;						
0.7	12/3/*h*1	*ADEJKLN*	⅓, 0, 1/12; 2$\sqrt{2}$				0.74048		
0.7′		*DEHIJLN*	⅓, 0, ¼;						
(0.7)		*EFHILMN*	⅓, 0, 5/12;						
1.1	6/3/*h*15	*BCDG*	⅓, $\frac{64}{141}-\frac{12}{47}\sqrt{2}$, 5/12, $\frac{16}{47}\sqrt{6}-\frac{18}{47}\sqrt{3}$				0.31648	$\frac{16}{47}\sqrt{6}-\frac{18}{47}\sqrt{3} \le c < \frac{2}{10129}[30387(137-24\sqrt{15})]^{1/2}$	
1.2	6/3/*h*1	*BDFG*	$\frac{140}{339}-\frac{2}{113}\sqrt{105}$, $\frac{70}{339}-\frac{1}{113}\sqrt{105}$, 5/12, $\frac{4}{113}\sqrt{105}-\frac{18}{113}$				0.45038	$\frac{4}{113}\sqrt{105}-\frac{18}{113} \le c < \frac{2}{10129}[30387(137-24\sqrt{15}]^{1/2}$	
1.3	7/3/*h*1	*ABDF*	$\frac{36}{73}-\frac{2}{73}\sqrt{105}$, $\frac{18}{73}-\frac{1}{73}\sqrt{105}$, ¼, $\frac{36}{73}-\frac{2}{73}\sqrt{105}$				0.49102	$\frac{36}{73}-\frac{2}{73}\sqrt{105} \le c < \frac{2}{4129}[12387(113-24\sqrt{15})]^{1/2}$	
1.4	7/3/*h*25	*ABCD*	0.24275, 0.04757, 0.03466; 0.22334				0.54283	$0.22334 \le c < \frac{2}{4129}[12387(113-24\sqrt{15})]^{1/2}$	
1.4′		*BCFG*							
1.5	7/4/*h*1	*BCDF*	0.24841, 0.05780, 0.41537; 0.22844				0.56791	$0.22844 \le c < \frac{2}{4129}[12387(113-24\sqrt{15})]^{1/2}$	
1.6	6/3/*h*45	*CDFG*	0.27784, 0.08585, 5/12; 0.42538				0.43125	$\frac{2}{10129}[30387(137-24\sqrt{15})]^{1/2} < c < \frac{1}{7}(54\sqrt{57}-390)^{1/2}$	
1.7	8/3/*h*8	*ABCF*	$\frac{27}{49}-\frac{12}{49}\sqrt{2}$, 0, ¼, $\frac{18}{49}\sqrt{3}-\frac{8}{49}\sqrt{6}$				0.60791	$\frac{18}{49}\sqrt{3}-\frac{8}{49}\sqrt{6} \le c < \frac{2}{4129}[12387(113-24\sqrt{15})]^{1/2}$	
1.8	7/3/*h*26	*ACDF*	0.21327, 0.02995, ¼; 0.34710				0.58208	$\frac{2}{4129}[12387(113-24\sqrt{15})]^{1/2} < c < \frac{2}{91}\sqrt{546}$	
1.9	7/3/*h*2	*ADEF*	$\frac{2}{11}$, $\frac{1}{11}$, ¼, $\frac{2}{11}\sqrt{6}$				0.49569	$\frac{2}{11}\sqrt{6} \le c < \frac{2}{91}\sqrt{546}$	
1.10	8/3/*h*9	*ACEF*	$\frac{9}{8}-\frac{1}{8}\sqrt{57}$, 0, ¼, $\frac{9}{4}\sqrt{2}-\frac{1}{4}\sqrt{114}$				0.65695	$\frac{9}{4}\sqrt{2}-\frac{1}{4}\sqrt{114} \le c < \frac{2}{91}\sqrt{546}$	
1.11	7/3/*h*23	*CEFH*	0.20519, 0.01855, 0.30844; 0.72560				0.59198	$\frac{2}{91}\sqrt{546} < c < \frac{1}{7}\sqrt{42}$	
1.11′		*ACDE*							
1.12	7/3/*h*24	*CDEF*	0.20816, 0.02319, 0.30955; 0.73360				0.59463	$\frac{2}{91}\sqrt{546} < c < \frac{1}{7}\sqrt{42}$	
1.13	7/3/*h*27	*CDFI*	0.31237, 0.11328, 0.38440; 0.69798				0.50096	$\frac{1}{7}(54\sqrt{57}-390)^{1/2} < c < \frac{1}{7}\sqrt{42}$	
1.14	6/4/*h*1	*DFGI*	⅓, ⅙, 5/12, ¼$\sqrt{6}$				0.51013	$\frac{1}{7}(54\sqrt{57}-390)^{1/2} < c \le \frac{1}{4}\sqrt{6}$	
1.15	6/3/*h*19	*CDEH*	0.20389, 0.01672, 0.29829; 0.72071				0.59132	$\frac{3}{2}\sqrt{3}-\frac{1}{2}\sqrt{15} \le c < \frac{1}{7}\sqrt{42}$	
1.16	7/3/*h*3	*DEFI*	0.24417, 0.12209, 0.34206; 0.94675				0.56478	$\frac{1}{7}\sqrt{42} < c \le 0.94675$	
1.17	6/3/*h*47	*CDHI*	0.30229, 0.02415, 0.28372; 1.03661				0.47115	$\frac{1}{7}\sqrt{42} < c < \frac{2}{3}\sqrt{3}$	
1.18	7/3/*h*29	*CFHI*	0.29960, 0.02573, 0.39421; 1.06124				0.49817	$\frac{1}{7}\sqrt{42} < c < \frac{2}{3}\sqrt{3}$	
1.19	6/3/*h*44	*DEHI*	0.25668, 0.04340, 0.27457; 1.62773				0.46814	$\frac{1}{7}\sqrt{42} < c < 2\sqrt{2}$	
1.20	7/3/*h*28	*EFHI*	0.25494, 0.04387, 0.39211; 1.54720				0.48091	$\frac{1}{7}\sqrt{42} < c < 2\sqrt{2}$	
1.21	6/4/*h*2	*ADJK*	⅓, 0, 1/12; $\sqrt{2}$				0.52360	$\frac{2}{3}\sqrt{3} < c < 2\sqrt{2}$	
1.21′		*DHIJ*	⅓, 0, ¼;						

Table 1 (continued)

(1.21)		*FHIM*	$\frac{1}{3}$, 0, $\frac{5}{12}$;		
1.22	8/3/*c*2	*EFIL*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{5}{12}$; $\sqrt{6}$	0.55536	$\sqrt{6} \le c < 2\sqrt{2}$
1.23	7/3/*h*4	*ADEL*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{6}\sqrt{6}-\frac{1}{3}$; $\frac{3}{2}+\frac{1}{2}\sqrt{6}$	0.49926	$\frac{3}{2}+\frac{1}{2}\sqrt{6} \le c < 2\sqrt{2}$
(1.23)		*DEIL*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{3}-\frac{1}{6}\sqrt{6}$;		
2.1	5/4/*h*14	*BCD*	$\frac{1}{3}$, $\frac{16}{15}-\frac{2}{5}\sqrt{6}$, $\frac{1}{6}$, $\frac{4}{5}\sqrt{6}-\frac{9}{5}$	0.27718	$\frac{4}{5}\sqrt{6}-\frac{9}{5} \le c < \frac{2}{4129}[12387(113-24\sqrt{15})]^{1/2}$
2.1′		*BCG*			
2.2	4/5/*h*1	*CDG*	$\frac{1}{3}$, 0.09720, $\frac{5}{12}$; 0.25508	0.28622	$\frac{16}{47}\sqrt{6}-\frac{18}{47}\sqrt{3} < c < \frac{1}{7}(54\sqrt{57}-390)^{1/2}$
2.3	5/4/*h*1	*ABD*	0.22360, 0.11180, 0.05625; 0.19488	0.41330	$0.19488 \le c < \frac{2}{4129}[12387(113-24\sqrt{15})]^{1/2}$
2.3′		*BFG*			
2.4	5/4/*h*2	*BDF*	0.22776, 0.11388, 0.38321; 0.20286	0.44786	$0.20286 \le c < \frac{2}{4129}[12387(113-24\sqrt{15})]^{1/2}$
2.5	4/6/*h*4	*DFG*	0.25523, 0.12762, $\frac{5}{12}$; 0.37082	0.35482	$\frac{4}{113}\sqrt{105}-\frac{18}{113} < c < \frac{1}{4}\sqrt{6}$
2.6	5/4/*h*3	*ADF*	0.20168, 0.10084, $\frac{1}{4}$; 0.31100	0.44750	$\frac{36}{73}-\frac{2}{73}\sqrt{105} < c < \frac{2}{91}\sqrt{546}$
2.7	6/4/*h*9	*ABC*	$\frac{9}{19}-\frac{2}{19}\sqrt{6}$, 0, 0; $\frac{9}{19}-\frac{2}{19}\sqrt{6}$	0.50701	$\frac{9}{19}-\frac{2}{19}\sqrt{6} \le c < \frac{2}{4129}[12387(113-24\sqrt{15})]^{1/2}$
2.7′		*BCF*			
2.8	5/3/*c*1	*ACD*	$\frac{5}{12}-\frac{1}{12}\sqrt{3}$, $\frac{1}{3}-\frac{1}{6}\sqrt{3}$, $\frac{1}{3}-\frac{1}{6}\sqrt{3}$; $\frac{1}{4}\sqrt{6}$	0.30812	$0.22334 < c < \frac{2}{3}\sqrt{3}$
2.8′		*CFG*			
2.8″		*CFH*			
2.8‴		*CDI*			
2.9	5/4/*h*38	*CDF*	0.26615, 0.07325, 0.38239; 0.46960	0.41918	$0.22844 < c < \frac{1}{7}\sqrt{42}$
2.10	6/4/*h*8	*ACF*	0.19854, 0, $\frac{1}{4}$; 0.32962	0.56721	$\frac{18}{49}\sqrt{3}-\frac{8}{49}\sqrt{6} < c < \frac{2}{91}\sqrt{546}$
2.11	5/3/*c*3	*ADE*	$\frac{2}{3}-\frac{1}{3}\sqrt{2}$, $\frac{1}{3}-\frac{1}{6}\sqrt{2}$, $\frac{1}{3}-\frac{1}{6}\sqrt{2}$; $\frac{1}{2}\sqrt{6}$	0.22327	$\frac{2}{11}\sqrt{6} < c < 2\sqrt{2}$
2.11′		*EFH*			
2.12	6/3/*h*46	*CEF*	{0.1967, 0.01322, 0.29279; 0.65}	>0.59198	$\frac{9}{4}\sqrt{2}-\frac{1}{4}\sqrt{114} < c < \frac{1}{7}\sqrt{42}$
2.12′		*ACE*			
2.13	5/3/*h*21	*CDE*	{0.20301, 0.01824, 0.3; 0.7}	>0.59132	$\frac{2}{91}\sqrt{546} < c < \frac{1}{7}\sqrt{42}$
2.13′		*CEH*			
2.14	5/4/*h*4	*DFI*	0.30806, 0.15403, 0.38151; 0.73426	0.47896	$\frac{1}{7}(54\sqrt{57}-390)^{1/2} < c < 0.94675$
2.15	6/3/*h*16	*CFI*	$\frac{1}{3}$, 0.08820, $\frac{5}{12}$; 0.89096	0.45502	$\frac{1}{7}(54\sqrt{57}-390)^{1/2} < c < \frac{2}{3}\sqrt{3}$
2.16	4/5/*h*2	*CDH*	0.26212, 0, $\frac{1}{4}$; 0.90800	0.41845	$\frac{3}{2}\sqrt{3}-\frac{1}{2}\sqrt{15} < c < \frac{2}{3}\sqrt{3}$
2.17	4/3/*h*2	*DEH*	$\frac{5}{4}-\frac{1}{4}\sqrt{17}$, 0, $\frac{1}{4}$, $\frac{1}{2}(18\sqrt{17}-66)^{1/2}$	0.41571	$\frac{3}{2}\sqrt{3}-\frac{1}{2}\sqrt{15} < c < 2\sqrt{2}$
2.18	5/4/*h*5	*ACK*	$\frac{1}{3}$, 0, 0; 1	0.40307	$\frac{1}{7}\sqrt{42} < c < \frac{2}{3}\sqrt{3}$
2.18′		*CDJ*	$\frac{1}{3}$, 0, $\frac{1}{6}$;		
(2.18)		*CHI*	$\frac{1}{3}$, 0, $\frac{1}{3}$;		
2.19	4/6/*h*5	*DHI*	0.28910, 0.03128, 0.27441; 1.38646	0.43990	$\frac{1}{7}\sqrt{42} < c < 2\sqrt{2}$
2.20	5/3/*h*1	*DEI*	0.26686, 0.13343, 0.27603; 1.75278	0.39824	$\frac{1}{7}\sqrt{42} < c < 2\sqrt{2}$
2.21	5/4/*h*39	*FHI*	0.28147, 0.03476, 0.39683; 1.40024	0.47030	$\frac{1}{7}\sqrt{42} < c < 2\sqrt{2}$
2.22	6/3/*h*2	*EFI*	0.26526, 0.13263, 0.39556; 1.55532	0.44076	$\frac{1}{7}\sqrt{42} < c < 2\sqrt{2}$
*i*2.1	$c[5/3/c3]^2$	*DEF*	$\frac{2}{3}-\frac{1}{3}\sqrt{2}$, $\frac{1}{3}-\frac{1}{6}\sqrt{2}$, $\frac{1}{3}\sqrt{2}-\frac{1}{6}$; $\frac{1}{4}\sqrt{6}$	0.44653	$\frac{2}{11}\sqrt{6} < c < 0.94675$
3.1	3/8/*h*1	*CD*	$\frac{1}{3}$, $\frac{19}{24}-\frac{1}{8}\sqrt{33}$, $\frac{1}{6}$, $\frac{9}{8}-\frac{1}{8}\sqrt{33}$	0.17248	$\frac{4}{5}\sqrt{6}-\frac{9}{5} < c < \frac{2}{3}\sqrt{3}$
3.1′		*CH*			
(3.1′)		*CG*			
3.2	3/6/*h*1	*AD*	0.23974, 0.11987, 0.06506; 0.83119	0.16799	$0.19488 < c < 2\sqrt{2}$
3.2′		*FG*			
(3.2)		*DI*			
(3.2′)		*FH*			
3.3	4/4/*c*1	*AC*	$\frac{1}{4}$, 0, 0; $\frac{1}{4}\sqrt{6}$	0.27768	$\frac{9}{19}-\frac{2}{19}\sqrt{6} < c < \frac{2}{3}\sqrt{3}$
3.3′		*CF*			
(3.3)		*CI*			
3.4	4/6/*c*2	*FI*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{5}{12}$; $\frac{1}{2}\sqrt{6}$	0.39270	$\frac{1}{7}(54\sqrt{57}-390)^{1/2} < c < 2\sqrt{2}$
*i*3.1	$h[3/6/h1]^2$	*DF*	0.23974, 0.11987, 0.36987; 0.41559	0.33598	$0.20286 < c < 0.94675$
*i*3.2	$h[4/3/h1]^2$	*CE*	0.20189, 0.016, 0.3; 0.7	>0.59132	$\frac{9}{4}\sqrt{2}-\frac{1}{4}\sqrt{114} < c < \frac{1}{7}\sqrt{42}$

P*312 6*l **$0 < x \le \frac{1}{3}$; $0 \le y \le \frac{1}{2}x$; $0 \le z \le \frac{1}{4}$**

A	$x, x-y, -z$		*C*	$-y, -x, -z$	*E*	$x, x-y, -z+1$	*I*	$-y+1, x-y, z$
					F	$-x+y+1, y, -z$		$-x+y+1, -x+1, z$
B	$x, y, z+1$		*D*	$-y, x-y, z$	*G*	$-y, -x, -z+1$	*J*	$-y, x-y-1, z$
	$x, y, z-1$			$-x+y, -x, z$	*H*	$-x+y+1, y, -z+1$		$-x+y+1, -x, z$
0.1	8/3/*h*3	*ABCEFGH*	$\frac{1}{3}$, 0, $\frac{1}{4}$, $\frac{2}{9}\sqrt{3}$				0.53742	
0.2	12/3/*h*1	*ACDEFGHIJ*	$\frac{1}{3}$, 0, $\frac{1}{4}$, $\frac{2}{3}\sqrt{2}$				0.74048	
1.1	5/4/*h*5	*ABCF*	$\frac{1}{3}$, 0, 0; $\frac{1}{3}$				0.40307	$\frac{1}{3} \le c < \frac{2}{9}\sqrt{3}$
1.2	6/4/*h*2	*ACEFGH*	$\frac{1}{3}$, 0, $\frac{1}{4}$; $\frac{1}{3}\sqrt{2}$				0.52360	$\frac{2}{9}\sqrt{3} < c < \frac{2}{3}\sqrt{2}$
1.3	6/3/*h*13	*ADEI*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{4}$; 1				0.45345	$\frac{2}{3}\sqrt{2} < c \le 1$

P*321 6*g **0, $2x-1 \le y \le \frac{1}{2}x$; $0 \le z \le \frac{1}{4}$**

A	$x-y, -y, -z$	*C*	$-y+1, x-y, z$	*E*	$x-y, -y, -z+1$	*H*	$-y, x-y-1, z$
			$-x+y+1, -x+1, z$	*F*	$y, x, -z$		$-x+y+1, -x, z$

Table 1 (continued)

B	$x, y, z+1$		*D*	$-y, x-y, z$	*G*	$y, x, -z+1$		
	$x, y, z-1$			$-x+y, -x, z$				
0.1	8/3/*h*11	*ABCEFG*	$\frac{2}{5}, \frac{1}{5}, \frac{1}{4}, \frac{2}{5}$				0.58042	
0.2	8/3/*h*10	*ACDEFG*	$\frac{1}{3}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}\sqrt{6}$				0.55536	
0.3	8/3/*h*4	*ACDEH*	$\frac{1}{3}, 0, \frac{1}{4}, \frac{2}{3}\sqrt{3}$				0.60460	
1.1	6/3/*h*21	*ABCE*	$\frac{4}{7}, \frac{1}{7}, \frac{1}{4}, \frac{2}{7}$				0.29613	$\frac{2}{7} \leq c < \frac{2}{5}$
1.2	6/3/*h*20	*ABCF*	$1-\frac{1}{3}\sqrt{3}, \frac{1}{2}-\frac{1}{6}\sqrt{3}, 0; \frac{1}{2}\sqrt{3}-\frac{1}{2}$				0.48601	$\frac{1}{2}\sqrt{3}-\frac{1}{2} \leq c < \frac{2}{5}$
1.3	6/3/*h*22	*ACEFG*	$\frac{13}{12}-\frac{1}{12}\sqrt{73}, \frac{13}{24}-\frac{1}{24}\sqrt{73}, \frac{1}{4}; \frac{1}{6}(39-3\sqrt{73})^{1/2}$				0.51755	$\frac{2}{5} < c < \frac{1}{3}\sqrt{6}$
1.4	6/3/*h*27	*ACDE*	$\frac{1}{3}, \frac{4}{15}-\frac{1}{15}\sqrt{6}, \frac{1}{4}; \frac{2}{15}(15+15\sqrt{6})^{1/2}$				0.50729	$\frac{1}{3}\sqrt{6} < c < \frac{2}{3}\sqrt{3}$
1.5	6/3/*h*13	*ACEH*	$\frac{1}{2}, 0, \frac{1}{4}; 1$				0.45345	$1 \leq c < \frac{2}{3}\sqrt{3}$
2.1	5/3/*h*5	*ABC*	$\frac{1}{3}\sqrt{3}, \frac{2}{3}\sqrt{3}-1, 0; 2-\sqrt{3}$				0.26045	$2-\sqrt{3} \leq c < \frac{2}{5}$
2.2	4/3/*h*3	*ACE*	$\frac{1}{6}\sqrt{19}-\frac{1}{6}, \frac{1}{3}\sqrt{19}-\frac{4}{3}, \frac{1}{4}; \frac{1}{3}(6\sqrt{19}-24)^{1/2}$				0.24427	$\frac{2}{7} < c < \frac{2}{3}\sqrt{3}$
P*3$_2$12 6*c			**$0 < x \leq \frac{1}{3}$; $0 \leq y \leq \frac{1}{2}x$; $0 \leq z < \frac{1}{2}$**					
A	$x, x-y, -z$		*D*	$-x+y, y, -z+\frac{2}{3}$	*H*	$x+1, y, z$	$x-1, y, z$	
B	$x, y, z+1$		*E*	$x, x-y, -z+1$		$x, y+1, z$	$x, y-1, z$	
	$x, y, z-1$		*F*	$-x+y+1, y, -z+\frac{2}{3}$		$x+1, y+1, z$	$x-1, y-1, z$	
C	$-y, -x, -z-\frac{1}{3}$		*G*	$-y+1, -x+1, -z+\frac{1}{3}$				
0.1	6/3/*h*17	*ABCEF*	$\frac{1}{3}, \frac{97}{339}-\frac{3}{113}\sqrt{105}, \frac{1}{4}; \frac{12}{113}\sqrt{35}-\frac{18}{113}\sqrt{3}$				0.45038	
0.2	6/4/*h*3	*ACDEFG*	$\frac{1}{3}, \frac{1}{6}, \frac{1}{4}; \frac{3}{4}\sqrt{2}$				0.51013	
0.3	10/3/*h*3	*CDFGH*	$\frac{1}{3}, \frac{1}{6}, \frac{1}{4}; 3\sqrt{3}$				0.69813	
1.1	5/4/*h*15	*BCEF*	$\frac{16}{23}-\frac{6}{23}\sqrt{2}, 0, \frac{1}{3}; \frac{12}{23}\sqrt{2}-\frac{9}{23}$				0.43565	$\frac{12}{23}\sqrt{2}-\frac{9}{23} \leq c < \frac{12}{113}\sqrt{35}-\frac{18}{113}\sqrt{3}$
1.2	4/6/*h*3	*ACEF*	$\frac{1}{3}$, 0.04951, $\frac{1}{4}$; 0.64227				0.35482	$\frac{12}{113}\sqrt{35}-\frac{18}{113}\sqrt{3} < c < \frac{3}{4}\sqrt{2}$
1.3	4/6/*h*1	*CDFG*	$\frac{1}{3}, \frac{1}{6}, \frac{1}{4}; \frac{3}{2}\sqrt{2}$				0.39270	$\frac{3}{4}\sqrt{2} < c < 3\sqrt{3}$
1.4	4/4/*h*1	*CDEF*	$\frac{1}{4}, 0, \frac{1}{3}; \frac{3}{4}\sqrt{3}$				0.34907	$\frac{3}{4}\sqrt{2} < c \leq \frac{3}{4}\sqrt{3}$
1.5	9/3/*h*3	*DEFH*	$\frac{1}{3}, \frac{1}{6}, \frac{1}{3}\sqrt{3}-\frac{1}{6}; 3+\frac{3}{2}\sqrt{3}$				0.64801	$3\sqrt{3} < c \leq 3+\frac{3}{2}\sqrt{3}$
1.5′		*ACGH*	$\frac{1}{3}, \frac{1}{6}, \frac{2}{3}-\frac{1}{3}\sqrt{3}$;					
1.5″		*CDFH*						
2.1	3/12/*h*1	*CEF*	$\frac{17}{24}-\frac{1}{24}\sqrt{97}, 0, \frac{1}{3}; \frac{1}{8}(102-6\sqrt{97})^{1/2}$				0.29229	$\frac{12}{23}\sqrt{2}-\frac{9}{23} < c < \frac{3}{4}\sqrt{3}$
2.2	3/10/*h*1	*DEF*	$\frac{1}{3}, \frac{1}{6}, \frac{3}{8}; \frac{3}{2}\sqrt{2}$				0.25507	$\frac{3}{4}\sqrt{2} < c < 3+\frac{3}{2}\sqrt{3}$
2.2′		*ACG*	$\frac{1}{3}, \frac{1}{6}, \frac{1}{8}$;					
2.2″		*CDF*						
2.3	8/3/*h*4	*CDH*	$0, 0, \frac{1}{4}; 6$				0.60460	$3\sqrt{3} < c \leq 6$
(2.3)		*ACH*	$0, 0, \frac{1}{12}$;					
(2.3)		*DEH*	$0, 0, \frac{5}{12}$;					
2.3′		*CFH*						
2.3″		*EFH*						
P*3$_2$21 6*c			**$0, 2x-1 \leq y \leq \frac{1}{2}x$; $0 \leq z < \frac{1}{2}$**					
A	$y, x, -z$		*F*	$y, x, -z+1$	*J*	$y, x-1, -z$	*M*	$x+1, y, z$
B	$x-y, -y, \frac{1}{3}-z$		*G*	$x-y, -y, -z+\frac{4}{3}$		$y+1, x, -z$		$x-1, y, z$
C	$x, y, z+1$		*H*	$-x+1, -x+y+1, -z+\frac{2}{3}$	*K*	$x-y+1, -y+1, -z+\frac{1}{3}$		$x, y+1, z$
	$x, y, z-1$			$-x+1, -x+y, -z+\frac{2}{3}$		$x-y, -y+1, -z+\frac{1}{3}$		$x, y-1, z$
D	$-y+1, x-y, z-\frac{1}{3}$		*I*	$y, x-1, -z+1$	*L*	$-x+2, -x+y+1, -z+\frac{2}{3}$		$x+1, y+1, z$
	$-x+y+1, -x+1, z+\frac{1}{3}$			$y+1, x, -z+1$				$x-1, y-1, z$
E	$-x, -x+y, -z+\frac{2}{3}$							
0.1	7/3/*h*30	*BCDFG*	$\frac{1}{15}(5-4\sqrt{5}+\sqrt{105}), \frac{1}{10}(15+9\sqrt{5}-5\sqrt{21}-\sqrt{105}), \frac{5}{12}$; $\frac{1}{5}(15+9\sqrt{5}-5\sqrt{21}-\sqrt{105})$				0.56016	
0.2	8/3/*h*6	*BDGHI*	$\frac{15}{14}-\frac{1}{14}\sqrt{57}, \frac{8}{7}-\frac{1}{7}\sqrt{57}, \frac{5}{12}; \frac{3}{7}(18\sqrt{57}-130)^{1/2}$				0.52528	
0.3	7/3/*h*9	*BDFGH*	$\frac{8}{7}-\frac{4}{21}\sqrt{15}, \frac{2}{7}-\frac{1}{21}\sqrt{15}, \frac{5}{12}; \frac{12}{7}\sqrt{3}-\frac{6}{7}\sqrt{5}$				0.58843	
0.4	8/4/*c*1	*BEFGHI*	$\frac{1}{3}, 0, \frac{5}{12}; \frac{1}{2}\sqrt{6}$				0.68017	
0.5	8/3/*h*5	*DFGHI*	$\frac{5}{16}+\frac{1}{48}\sqrt{33}, \frac{1}{48}\sqrt{33}-\frac{1}{48}, \frac{5}{24}+\frac{1}{24}\sqrt{33}; \frac{15}{16}+\frac{1}{16}\sqrt{33}$				0.58365	
0.6	8/3/*h*3	*ABDJK*	$\frac{2}{3}, \frac{1}{3}, \frac{1}{12}; 2$				0.53742	
(0.6)		*DFHIL*	$\frac{2}{3}, \frac{1}{3}, \frac{5}{12}$;					
0.6′		*BDHKL*	$\frac{2}{3}, \frac{1}{3}, \frac{1}{4}$;					
0.7	12/3/*h*1	*ABJKM*	$\frac{2}{3}, \frac{1}{3}, \frac{1}{12}; 2\sqrt{6}$				0.74048	
(0.7)		*FHILM*	$\frac{2}{3}, \frac{1}{3}, \frac{5}{12}$;					
0.7′		*BHKLM*	$\frac{2}{3}, \frac{1}{3}, \frac{1}{4}$;					
0.8	12/3/*c*1	*EFHIM*	$\frac{1}{3}, 0, \frac{5}{12}; 2\sqrt{6}$				0.74048	
1.1	6/3/*h*12	*BCDG*	$\frac{10}{47}+\frac{12}{47}\sqrt{2}, \frac{24}{47}\sqrt{2}-\frac{27}{47}, \frac{5}{12}; \frac{48}{47}\sqrt{2}-\frac{54}{47}$				0.31648	$\frac{48}{47}\sqrt{2}-\frac{54}{47} \leq c < \frac{1}{5}(15+9\sqrt{5}-5\sqrt{21}-\sqrt{105})$
1.2	6/4/*h*6	*ABCD*	$\frac{70}{73}-\frac{8}{219}\sqrt{210}, \frac{35}{73}-\frac{4}{219}, \frac{1}{12}; \frac{6}{73}\sqrt{105}-\frac{24}{73}\sqrt{2}$				0.51632	$\frac{6}{73}\sqrt{105}-\frac{24}{73}\sqrt{2} \leq c < \frac{1}{5}(15+9\sqrt{5}-5\sqrt{21}-\sqrt{105})$
1.2′		*CDFG*						
1.3	6/4/*h*5	*BCDF*	$1-\frac{1}{3}\sqrt{3}, \frac{1}{2}-\frac{1}{6}\sqrt{3}, \frac{1}{3}; \frac{3}{8}\sqrt{6}-\frac{3}{8}\sqrt{2}$				0.54676	$\frac{3}{8}\sqrt{6}-\frac{3}{8}\sqrt{2} \leq c < \frac{1}{5}(15+9\sqrt{5}-5\sqrt{21}-\sqrt{105})$
1.4	5/4/*h*10	*BDFG*	0.41577, 0.16945, $\frac{5}{12}$; 0.65685				0.47192	$\frac{1}{5}(15+9\sqrt{5}-5\sqrt{21}-\sqrt{105}) < c < \frac{12}{7}\sqrt{3}-\frac{6}{7}\sqrt{5}$
1.5	6/3/*h*9	*BDGH*	0.50386, 0.06772, $\frac{5}{12}$; 1.03116				0.52006	$1.02944 \leq c < \frac{12}{7}\sqrt{3}-\frac{6}{7}\sqrt{5}$
1.6	6/4/*h*3	*BGHI*	$\frac{1}{2}, 0, \frac{5}{12}; \frac{3}{4}\sqrt{2}$				0.51013	$\frac{3}{7}(18\sqrt{57}-130)^{1/2} < c < \frac{1}{2}\sqrt{6}$

Table 1 (continued)

1.7	7/3/*h*10	*DGHI*	0.50963, 0.06871, 0.42329; 1.07436	0.52089	$\frac{3}{7}(18\sqrt{57}-130)^{1/2} < c < \frac{15}{16}+\frac{1}{16}\sqrt{33}$
1.8	6/3/*h*3	*BDFH*	$1-\frac{1}{3}\sqrt{3}, \frac{1}{2}-\frac{1}{6}\sqrt{3}, \frac{1}{3}, \frac{3}{2}\sqrt{3}-\frac{3}{2}$	0.45821	$\frac{12}{7}\sqrt{3}-\frac{6}{7}\sqrt{5} < c \leq \frac{3}{2}\sqrt{3}-\frac{3}{2}$
1.9	5/4/*h*8	*BFGH*	{0.38825, 0.07869, $\frac{5}{12}$; 1.1}	>0.58843	$\frac{12}{7}\sqrt{3}-\frac{6}{7}\sqrt{5} < c < \frac{1}{2}\sqrt{6}$
1.10	6/3/*h*8	*DFGH*	0.41973, 0.10081, 0.43589; 1.18193	0.57681	$\frac{12}{7}\sqrt{3}-\frac{6}{7}\sqrt{5} < c < \frac{15}{16}+\frac{1}{16}\sqrt{33}$
1.11	6/4/*h*4	*FGHI*	{0.35584, 0.02251, 0.42658; 1.26}	>0.58365	$\frac{1}{2}\sqrt{6} < c \leq 1.29904$
1.12	5/4/*h*11	*BEFH*	$\frac{1}{3}, \frac{1}{6}, \frac{1}{3}, \frac{3}{2}$	0.46542	$\frac{1}{2}\sqrt{6} < c \leq \frac{3}{2}$
1.13	6/4/*c*1	*EFHI*	$\frac{1}{3}$, 0, $\frac{5}{12}$; $\sqrt{6}$	0.52360	$\frac{1}{2}\sqrt{6} < c < 2\sqrt{6}$
1.14	7/3/*h*11	*DFHI*	0.55664, 0.22331, 0.43412; 1.77996	0.49321	$\frac{15}{16}+\frac{1}{16}\sqrt{33} < c < 2$
1.15	6/3/*h*11	*ABDJ*	0.55196, 0.21862, 0.03138; 1.65587	0.44226	$\frac{15}{16}+\frac{1}{16}\sqrt{33} < c < 2$
1.15′		*DFGI*			
1.16	6/4/*h*2	*ABJK*	$\frac{2}{5}, \frac{1}{3}, \frac{1}{12}$; $\sqrt{6}$	0.52360	$2 < c < 2\sqrt{6}$
(1.16)		*FHIL*	$\frac{2}{5}, \frac{1}{3}, \frac{5}{12}$;		
1.16′		*BHKL*	$\frac{2}{3}, \frac{1}{3}, \frac{1}{4}$;		
1.17	11/3/*h*2	*FHIM*	$\frac{1}{2}, \frac{1}{6}, \frac{11}{6}-\sqrt{2}; \frac{3}{2}\sqrt{3}+\sqrt{6}$	0.71868	$2\sqrt{6} < c \leq \frac{3}{2}\sqrt{3}+\sqrt{6}$
1.18	10/3/*h*4	*ABJM*	$\frac{1}{3}$, 0, $\frac{1}{6}\sqrt{6}-\frac{1}{3}$; $3+\sqrt{6}$	0.66568	$2\sqrt{6} < c \leq 3+\sqrt{6}$
(1.18)		*BEHM*	$\frac{1}{3}$, 0, $\frac{2}{3}-\frac{1}{6}\sqrt{6}$;		
1.18′		*EFHM*			
2.1	5/4/*h*13	*BCD*	$\frac{2}{5}\sqrt{6}-\frac{2}{5}, \frac{4}{5}\sqrt{6}-\frac{9}{5}, \frac{1}{6}, \frac{12}{5}\sqrt{2}-\frac{9}{5}\sqrt{3}$	0.27718	$\frac{12}{5}\sqrt{2}-\frac{9}{5}\sqrt{3} \leq c < \frac{1}{5}(15+9\sqrt{5}-5\sqrt{21}-\sqrt{105})$
(2.1)		*CDF*			
2.1′		*CDG*			
(2.1′)		*ACD*			
2.2	4/5/*h*4	*BDG*	0.56947, 0.13894, $\frac{5}{12}$; 0.44182	0.28622	$\frac{48}{47}\sqrt{2}-\frac{54}{47} < c < \frac{12}{7}\sqrt{3}-\frac{6}{7}\sqrt{5}$
2.3	4/5/*h*3	*ABD*	0.47297, 0.23648, $\frac{1}{12}$; 1.00012	0.31367	$\frac{6}{73}\sqrt{105}-\frac{24}{73}\sqrt{2} < c < 2$
2.3′		*DFG*			
2.4	4/4/*h*2	*BDF*	$1-\frac{1}{3}\sqrt{3}, \frac{1}{2}-\frac{1}{6}\sqrt{3}, \frac{1}{3}, \frac{3}{4}\sqrt{6}-\frac{3}{4}\sqrt{2}$	0.42089	$\frac{3}{8}\sqrt{6}-\frac{3}{8}\sqrt{2} < c < \frac{3}{2}\sqrt{3}-\frac{3}{2}$
2.5	5/3/*h*4	*BDH*	0.53614, 0.19783, 0.30502; 1.36410	0.35233	$1.02983 \leq c < 2$
2.5′		*DGI*			
2.5″		*DFH*			
(2.5″)		*BDJ*			
2.6	4/6/*h*14	*BGH*	{0.4, 0.06086, $\frac{5}{12}$; 1.1}	>0.51013	$1.02944 < c < \frac{1}{2}\sqrt{6}$
2.7	5/4/*h*9	*DGH*	0.50383, 0.06815, 0.41816; 1.04015	0.52001	$1.02944 < c < \frac{15}{16}+\frac{1}{16}\sqrt{33}$
2.8	6/3/*h*10	*DHI*	0.57846, 0.15693, $\frac{5}{12}$; 1.54319	0.45502	$\frac{3}{7}(18\sqrt{57}-130)^{1/2} < c < 2$
2.9	5/4/*h*6	*GHI*	{0.44592, 0.01460, 0.42; 1.1}	>0.51013	$\frac{3}{7}(18\sqrt{57}-130)^{1/2} < c < 1.29904$
2.10	4/4/*h*3	*BFH*	$\frac{5}{8}-\frac{1}{24}\sqrt{33}, \frac{5}{16}-\frac{1}{48}\sqrt{33}, \frac{1}{3}, \frac{3}{16}+\frac{3}{16}\sqrt{33}$	0.44621	$\frac{12}{7}\sqrt{3}-\frac{6}{7}\sqrt{5} < c < \frac{3}{2}$
2.11	4/6/*h*6	*FGH*	{0.39518, 0.08, 0.42521; 1.15}	>0.57681	$\frac{12}{7}\sqrt{3}-\frac{6}{7}\sqrt{5} < c < 1.29904$
2.12	4/6/*c*1	*BEH*	$\frac{1}{3}$, 0, $\frac{7}{24}$; $\sqrt{6}$	0.34009	$\frac{1}{2}\sqrt{6} < c < 3+\sqrt{6}$
(2.12)		*ABJ*	$\frac{1}{3}$, 0, $\frac{1}{24}$;		
2.12′		*FGI*			
2.12″		*EFH*			
2.13	5/4/*h*12	*FHI*	$\frac{1}{2}, \frac{1}{6}$, 0.42812; 2.33585	0.46271	$\frac{1}{2}\sqrt{6} < c < \frac{3}{2}\sqrt{3}+\sqrt{6}$
2.14	5/4/*h*5	*ADJ*	$\frac{2}{3}, \frac{1}{3}$, 0; $\sqrt{3}$	0.40307	$\frac{15}{16}+\frac{1}{16}\sqrt{33} < c < 2$
(2.14)		*DHL*	$\frac{2}{3}, \frac{1}{3}, \frac{1}{3}$;		
2.14′		*BDK*	$\frac{2}{3}, \frac{1}{3}, \frac{1}{6}$;		
(2.14′)		*DFI*			
2.15	9/3/*h*3	*BJM*	$\frac{1}{2}$, 0, $\frac{1}{3}\sqrt{3}-\frac{1}{2}$; $3+\frac{3}{2}\sqrt{3}$	0.64801	$2\sqrt{6} < c \leq 3+\frac{3}{2}\sqrt{3}$
(2.15)		*BHM*	$\frac{1}{2}$, 0, $\frac{5}{6}-\frac{1}{3}\sqrt{3}$;		
(2.15)		*FHM*			
2.16	8/3/*h*4	*ABM*	0, 0, $\frac{1}{12}$; 6	0.60460	$2\sqrt{6} < c \leq 6$
(2.16)		*BEM*	0, 0, $\frac{1}{4}$;		
(2.16)		*EFM*	0, 0, $\frac{5}{12}$;		
2.17	10/3/*h*3	*HIM*	$\frac{1}{2}$, 0, $\frac{5}{12}$; $3\sqrt{3}$	0.69813	$2\sqrt{6} < c \leq 3\sqrt{3}$
3.1	3/8/*h*2	*BD*	$\frac{1}{8}\sqrt{33}-\frac{1}{8}, \frac{1}{4}\sqrt{33}-\frac{5}{4}, \frac{1}{6}, \frac{9}{8}\sqrt{3}-\frac{3}{8}\sqrt{11}$	0.17248	$\frac{12}{5}\sqrt{2}-\frac{9}{5}\sqrt{3} < c < 2$
(3.1)		*AD*			
(3.1)		*DF*			
3.1′		*DG*			
3.2	4/4/*h*4	*DH*	$\frac{1}{8}+\frac{1}{24}\sqrt{105}, \frac{1}{16}+\frac{1}{48}\sqrt{105}, \frac{1}{3}, \frac{3}{16}\sqrt{3}+\frac{3}{16}\sqrt{35}$	0.33170	$1.02944 < c < 2$
(3.2)		*DJ*			
3.2′		*DI*			
3.3	3/10/*h*1	*BH*	$\frac{1}{2}$, 0, $\frac{7}{24}$; $\frac{3}{2}\sqrt{2}$	0.25507	$1.02983 < c < 3+\frac{3}{2}\sqrt{3}$
(3.3)		*BJ*	$\frac{1}{2}$, 0, $\frac{1}{24}$;		
(3.3)		*FH*			
3.3′		*GI*			
3.4	4/6/*h*1	*HI*	$\frac{1}{2}$, 0, $\frac{5}{12}$; $\frac{3}{2}\sqrt{2}$	0.39270	$\frac{3}{7}(18\sqrt{57}-130)^{1/2} < c < 3\sqrt{3}$
*i*3.1	$h[3/10/h1]^2$	*GH*	{0.41, 0.07341, 0.42; 1.1}	>0.51013	$1.02944 < c < 1.29904$

Table 1 (continued)

R*32 18*f			**$0 < x \le \frac{1}{3}$; $0 \le y \le \frac{1}{2}x$; $0 \le z < \frac{1}{2}$**				
A	$x-y, -y, -z$		*E* $-y, x-y, z$	*I* $-x+\frac{2}{3}, -x+y+\frac{1}{3}, -z+\frac{4}{3}$	*L*	$-y, x-y-1, z$	
B	$y, x, -z$		$-x+y, -x, z$	$x-y+\frac{1}{3}, -y+\frac{2}{3}, -z+\frac{2}{3}$		$-x+y+1, -x, z$	
C	$-x+\frac{2}{3}, -x+y+\frac{1}{3}, -z+\frac{1}{3}$		*F* $x-y, -y, -z+1$	*K* $-y+1, x-y, z$			
D	$x, y, z+1$		*G* $y, x, -z+1$	$-x+y+1, -x+1, z$			
	$x, y, z-1$		*H* $y+\frac{1}{3}, x-\frac{1}{3}, -z+\frac{2}{3}$				
0.1	6/3/*h*1	*CDFGI*	$\frac{140}{339}-\frac{2}{113}\sqrt{105}, \frac{70}{339}-\frac{1}{113}\sqrt{105}, \frac{5}{12}; \frac{4}{113}\sqrt{105}-\frac{18}{113}$		0.45038		
0.2	7/3/*h*1	*ABCDFG*	$\frac{36}{73}-\frac{2}{73}\sqrt{105}, \frac{18}{73}-\frac{1}{73}\sqrt{105}, \frac{1}{4}; \frac{36}{73}-\frac{2}{73}\sqrt{105}$		0.49102		
0.3	7/3/*h*2	*ABCEFG*	$\frac{2}{11}, \frac{1}{11}, \frac{1}{4}; \frac{2}{11}\sqrt{6}$		0.49569		
0.4	6/4/*h*7	*ACEFH*	$3-2\sqrt{2}, 0, \frac{1}{4}; 6\sqrt{3}-4\sqrt{6}$		0.48054		
0.5	6/4/*h*1	*CFGHIJ*	$\frac{1}{3}, \frac{1}{6}, \frac{5}{12}; \frac{1}{4}\sqrt{6}$		0.51013		
0.6	7/3/*h*3	*CEFGHJ*	0.24417, 0.12209, 0.34206; 0.94675		0.56478		
0.7	8/3/*c*2	*EFGHJK*	$\frac{1}{3}, \frac{1}{6}, \frac{5}{12}; \sqrt{6}$		0.55536		
0.8	7/3/*h*4	*ABCEK*	$\frac{1}{3}, \frac{1}{6}, \frac{1}{6}\sqrt{6}-\frac{1}{3}; \frac{3}{2}+\frac{1}{2}\sqrt{6}$		0.49926		
(0.8)		*CEHJK*	$\frac{1}{3}, \frac{1}{6}, \frac{2}{3}-\frac{1}{6}\sqrt{6};$				
0.9	8/3/*h*4	*ACEKL*	$\frac{1}{3}, 0, \frac{1}{12}; 2\sqrt{3}$		0.60460		
(0.9)		*CEHKL*	$\frac{1}{3}, 0, \frac{1}{4};$				
(0.9)		*EFHKL*	$\frac{1}{3}, 0, \frac{5}{12};$				
1.1	5/4/*h*1	*ABCD*	0.22360, 0.11180, 0.05625; 0.19488		0.41330	$0.19488 \le c < \frac{36}{73}-\frac{2}{73}\sqrt{105}$	
1.1′		*DFGI*					
1.2	5/4/*h*2	*CDFG*	0.22776, 0.11388, 0.38321; 0.20286		0.44786	$0.20286 \le c < \frac{36}{73}-\frac{2}{73}\sqrt{105}$	
1.3	4/6/*h*4	*CFGI*	0.25523, 0.12762, $\frac{5}{12}$; 0.37082		0.35482	$\frac{4}{113}\sqrt{105}-\frac{18}{113} < c < \frac{1}{4}\sqrt{6}$	
1.4	5/4/*h*3	*ABCFG*	0.20168, 0.10084, $\frac{1}{4}$; 0.31100		0.44750	$\frac{36}{73}-\frac{2}{73}\sqrt{105} < c < \frac{2}{11}\sqrt{6}$	
1.5	5/3/*c*3	*ABCE*	$\frac{2}{3}-\frac{1}{3}\sqrt{2}, \frac{1}{3}-\frac{1}{6}\sqrt{2}, \frac{1}{3}-\frac{1}{6}\sqrt{2}; \frac{1}{2}\sqrt{6}$		0.22327	$\frac{2}{11}\sqrt{6} < c < \frac{3}{2}+\frac{1}{2}\sqrt{6}$	
1.6	5/3/*h*22	*ACEF*	0.17988, 0.04565, $\frac{1}{4}$; 0.53829		0.44628	$\frac{2}{11}\sqrt{6} < c < 6\sqrt{3}-4\sqrt{6}$	
1.7	5/3/*h*23	*CEFH*	0.19342, 0.03552, 0.29585; 0.74137		0.43263	$6\sqrt{3}-4\sqrt{6} < c < 0.94675$	
1.8	5/4/*h*4	*CFGHJ*	0.30806, 0.15403, 0.38151; 0.73426		0.47896	$\frac{1}{4}\sqrt{6} < c < 0.94675$	
1.9	6/3/*h*2	*EFGHJ*	0.26526, 0.13263, 0.39556; 1.55532		0.44076	$0.94675 < c < \sqrt{6}$	
1.10	5/3/*h*1	*CEHJ*	0.26686, 0.13343, 0.27603; 1.75278		0.39824	$0.94675 < c < \frac{3}{2}+\frac{1}{2}\sqrt{6}$	
1.11	6/3/*h*18	*EFHK*	$\frac{1}{3}, \frac{4}{15}-\frac{1}{15}\sqrt{6}, \frac{5}{12}; \frac{2}{5}(15+15\sqrt{6})^{1/2}$		0.50729	$\sqrt{6} < c < 2\sqrt{3}$	
1.12	6/3/*h*48	*ACEK*	$\frac{1}{3}$, 0.12724, 0.07893; 2.87586		0.48632	$\frac{3}{2}+\frac{1}{2}\sqrt{6} < c < 2\sqrt{3}$	
(1.12)		*CEHK*	$\frac{1}{3}$, 0.12724, 0.25440;				
*i*1.1	$c[5/3/c3]^2$	*CEFG*	$\frac{2}{3}-\frac{1}{3}\sqrt{2}, \frac{1}{3}-\frac{1}{6}\sqrt{2}, \frac{1}{3}\sqrt{2}-\frac{1}{6}; \frac{1}{4}\sqrt{6}$		0.44653	$\frac{2}{11}\sqrt{6} < c < 0.94675$	
2.1	3/6/*h*1	*ABC*	0.23974, 0.11987, 0.06506; 0.83119		0.16799	$0.19488 < c < \frac{3}{2}+\frac{1}{2}\sqrt{6}$	
2.1′		*FGI*					
(2.1)		*CHJ*					
2.2	4/3/*h*5	*ACE*	0.19127, 0.05734, 0.10525; 1.31706		0.21101	$\frac{2}{11}\sqrt{6} < c < 2\sqrt{3}$	
2.2′		*EFH*					
2.3	4/3/*h*1	*CEH*	$\frac{1}{5}, 0, \frac{1}{4}; \frac{2}{5}\sqrt{15}$		0.29202	$6\sqrt{3}-4\sqrt{6} < c < 2\sqrt{3}$	
2.4	4/6/*c*2	*FGHJ*	$\frac{1}{3}, \frac{1}{6}, \frac{5}{12}; \frac{1}{2}\sqrt{6}$		0.39270	$\frac{1}{4}\sqrt{6} < c < \sqrt{6}$	
*i*2.1	$h[3/6/h1]^2$	*CFG*	0.23974, 0.11987, 0.36987; 0.41559		0.33598	$0.20286 < c < 0.94675$	
*i*2.2	$h[4/3/h5]^2_{\rm I}$	*CEF*	0.19127, 0.05734, 0.28951; 0.65853		0.42202	$\frac{2}{11}\sqrt{6} < c < 0.94675$	
P*$\bar{3}$1*m* 12*l			**$0 < y \le \frac{1}{2}x$; $2x-1 \le y$; $0 \le z \le \frac{1}{4}$**				
A	$x-y, -y, z$		*D* $x, y, z+1$	*E* $-x+y+1, -x+1, z$	*F*	$x, x-y, -z+1$	
B	$x, x-y, -z$		$x, y, z-1$	$-y+1, x-y, z$	*G*	$-x+y+1, y, -z+1$	
C	$-x+y+1, y, -z$				*H*	y, x, z	
0.1	7/3/*h*18	*ABCDFG*	$\frac{7}{15}, \frac{2}{15}, \frac{1}{4}; \frac{2}{15}\sqrt{3}$		0.38694		
0.2	7/3/*h*19	*ABCEFG*	$\frac{1}{2}, \frac{1}{6}, \frac{1}{4}; \frac{1}{3}\sqrt{2}$		0.37024		
0.3	6/3/*h*20	*ABEFH*	$1-\frac{1}{3}\sqrt{3}, \frac{1}{2}-\frac{1}{6}\sqrt{3}, \frac{1}{4}; \sqrt{3}-1$		0.48601		
1.1	5/4/*h*17	*ABCD*	$\frac{1}{6}+\frac{1}{6}\sqrt{3}, \frac{1}{6}\sqrt{3}-\frac{1}{6}, 0; \frac{1}{2}-\frac{1}{6}\sqrt{3}$		0.32400	$\frac{1}{2}-\frac{1}{6}\sqrt{3} \le c < \frac{2}{15}\sqrt{3}$	
1.2	5/4/*h*21	*ABCFG*	0.48100, 0.14767, $\frac{1}{4}$; 0.35182		0.34503	$\frac{2}{15}\sqrt{3} < c < \frac{1}{3}\sqrt{2}$	
1.3	5/3/*h*8	*ABEF*	0.48786, 0.17112, $\frac{1}{4}$; 0.51627		0.36587	$\frac{1}{3}\sqrt{2} < c < \sqrt{3}-1$	
1.4	5/3/*h*5	*ACEG*	$\frac{1}{3}\sqrt{3}, \frac{2}{3}\sqrt{3}-1, \frac{1}{4}; 4-2\sqrt{3}$		0.26045	$\frac{1}{3}\sqrt{2} < c \le 4-2\sqrt{3}$	
P*$\bar{3}$1*c* 12*i			**0, $2x-1 \le y \le \frac{1}{2}x$; $0 \le z \le \frac{1}{4}$**				
A	$x-y, x, -z$		*D* $x, x-y, -z+\frac{1}{2}$	*H* $-x+y+1, y, -z-\frac{1}{2}$	*K*	$-y, -x, -z+\frac{1}{2}$	
	$y, -x+y, -z$		*E* $x, x-y, -z-\frac{1}{2}$	*I* $x, y, z+1$	*L*	$-y, x-y-1, z$	
B	$x-y, -y, z+\frac{1}{2}$		*F* $-y+1, x-y, z$	$x, y, z-1$		$-x+y+1, -x, z$	
	$x-y, -y, z-\frac{1}{2}$		$-x+y+1, -x+1, z$	*J* $-y, x-y, z$			
C	$-x+1, -y, -z$		*G* $-x+y+1, y, -z+\frac{1}{2}$	$-x+y, -x, z$			
0.1	8/3/*h*15	*BDEGHI*	$\frac{1}{6}+\frac{1}{6}\sqrt{3}, \frac{1}{6}\sqrt{3}-\frac{1}{6}, 0; \frac{1}{3}\sqrt{3}-\frac{1}{3}$		0.43201		
0.2	7/3/*h*14	*BCDEGH*	$\frac{1}{6}+\frac{1}{6}\sqrt{3}, \frac{1}{6}\sqrt{3}-\frac{1}{6}, 0; 1-\frac{1}{3}\sqrt{3}$		0.45821		
0.3	7/3/*h*15	*CDEFGH*	$\frac{1}{3}\sqrt{6}-\frac{1}{3}, \frac{1}{3}\sqrt{6}-\frac{2}{3}, 0; 2\sqrt{2}-\frac{4}{3}\sqrt{3}$		0.44882		
0.4	7/3/*h*16	*ABCDE*	$\frac{1}{3}+\frac{1}{9}\sqrt{3}[2-(1+\sqrt{3})^{1/2}], \frac{1}{9}[\sqrt{3}+(3-2\sqrt{3})(1+\sqrt{3})^{1/2}], 0;$ $\frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}]$		0.54567		

Table 1 (continued)

0.5	7/3/*h*17	*ACDEF*	$\frac{3}{7}$, $\frac{1}{7}$, 0; $\frac{2}{7}\sqrt{6}$	0.55975	
0.6	8/3/*h*3	*ABCDGK*	$\frac{1}{3}$, 0, $\frac{1}{8}$, $\frac{4}{9}\sqrt{3}$	0.53742	
0.7	12/3/*h*1	*ACDFGJKL*	$\frac{1}{3}$, 0, $\frac{1}{8}$, $\frac{4}{3}\sqrt{2}$	0.74048	
1.1	6/3/*h*28	*BDGI*	$\frac{4}{9}$, $\frac{1}{9}$, $\frac{1}{4}$, $\frac{2}{9}$	0.35828	$\frac{2}{9} \le c < \frac{1}{3}\sqrt{3}-\frac{1}{3}$
1.2	6/4/*h*10	*BDEGH*	$\frac{1}{6}+\frac{1}{6}\sqrt{3}$, $\frac{1}{6}\sqrt{3}-\frac{1}{6}$, 0; $\frac{1}{2}\sqrt{2}-\frac{1}{6}\sqrt{6}$	0.42089	$\frac{1}{3}\sqrt{3}-\frac{1}{3} < c < 1-\frac{1}{3}\sqrt{3}$
1.3	5/4/*h*18	*BCDE*	{0.43301, 0.11603, 0; 0.5}	>0.45821	$1-\frac{1}{3}\sqrt{3} < c < \frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}]$
1.4	5/4/*h*19	*CDEGH*	0.47385, 0.14051, 0; 0.48675	0.44621	$1-\frac{1}{3}\sqrt{3} < c < 2\sqrt{2}-\frac{4}{3}\sqrt{3}$
1.5	5/3/*h*24	*BCDG*	$\frac{5}{12}$, $\frac{1}{12}$, $\frac{3}{4}-\frac{1}{4}\sqrt{7}$; $\frac{1}{6}(5+2\sqrt{7})^{1/2}$	0.38052	$1-\frac{1}{3}\sqrt{3} < c < \frac{4}{9}\sqrt{3}$
1.6	5/3/*h*7	*CDEF*	{0.47414, 0.14888, 0; 0.55}	>0.44882	$2\sqrt{2}-\frac{4}{3}\sqrt{3} < c < \frac{2}{7}\sqrt{6}$
1.7	5/3/*h*5	*CFGH*	$\frac{1}{3}\sqrt{3}$, $\frac{2}{3}\sqrt{3}-1$, 0; $4-2\sqrt{3}$	0.26045	$2\sqrt{2}-\frac{4}{3}\sqrt{3} < c \le 4-2\sqrt{3}$
1.8	5/3/*h*25	*CDFG*	0.46832, 0.13499, 0.08897; 0.87098	0.33776	$2\sqrt{2}-\frac{4}{3}\sqrt{3} < c < \frac{4}{3}\sqrt{2}$
1.9	5/4/*h*20	*ACDE*	0.40396, 0.11227, 0; 0.62688	0.54522	$\frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}] < c < \frac{2}{7}\sqrt{6}$
1.10	6/3/*h*49	*ABCD*	0.36389, 0.05442, 0.07052; 0.68092	0.46975	$\frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}] < c < \frac{4}{9}\sqrt{3}$
1.11	5/4/*h*5	*BDGK*	$\frac{1}{3}$, 0, $\frac{1}{4}$, $\frac{2}{3}$	0.40307	$\frac{2}{3} \le c < \frac{4}{9}\sqrt{3}$
1.12	6/3/*h*20	*ADEF*	$1-\frac{1}{3}\sqrt{3}$, $\frac{1}{2}-\frac{1}{6}\sqrt{3}$, 0; $\sqrt{3}-1$	0.48601	$\frac{2}{7}\sqrt{6} < c \le \sqrt{3}-1$
1.13	6/3/*h*50	*ACDF*	0.41447, 0.12572, 0.07751; 1.07076	0.44605	$\frac{2}{7}\sqrt{6} < c < \frac{4}{3}\sqrt{2}$
1.14	6/4/*h*2	*ACDGK*	$\frac{1}{3}$, 0, $\frac{1}{8}$, $\frac{2}{3}\sqrt{2}$	0.52360	$\frac{4}{9}\sqrt{3} < c < \frac{4}{3}\sqrt{2}$
1.15	7/3/*h*20	*ADFJ*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{4}\sqrt{6}-\frac{1}{2}$, $\frac{1}{3}\sqrt{6}+1$	0.49926	$\frac{1}{3}\sqrt{6}+1 \le c < \frac{4}{3}\sqrt{2}$
1.16	6/3/*h*13	*CFGL*	$\frac{1}{2}$, 0, $\frac{1}{8}$,; 2	0.45345	$\frac{4}{3}\sqrt{2} < c \le 2$
2.1	4/4/*h*5	*BDG*	$\frac{5}{12}$, $\frac{1}{12}$, $\frac{1}{4}$; $\frac{1}{6}\sqrt{6}$	0.27768	$\frac{2}{9} < c < \frac{4}{9}\sqrt{3}$
2.2	4/4/*h*6	*BCD*	{0.39501, 0.06497, 0.1; 0.6}	>0.38052	$1-\frac{1}{3}\sqrt{3} < c < \frac{4}{9}\sqrt{3}$
2.3	3/6/*h*2	*CDG*	0.44883, 0.11550, 0.09283; 0.78872	0.33065	$1-\frac{1}{3}\sqrt{3} < c < \frac{4}{3}\sqrt{2}$
2.4	4/3/*h*6	*CDF*	{0.47503, 0.14749, 0.04; 0.65}	>0.33776	$2\sqrt{2}-\frac{4}{3}\sqrt{3} < c < \frac{4}{3}\sqrt{2}$
2.5	4/3/*h*4	*CFG*	0.57018, 0.14036, 0.08796; 0.89321	0.19701	$2\sqrt{2}-\frac{4}{3}\sqrt{3} < c < 2$
2.6	5/4/*h*5	*ABC*	$\frac{1}{3}$, 0, 0; $\frac{2}{3}$	0.40307	$\frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}] < c < \frac{4}{9}\sqrt{3}$
2.7	4/6/*h*8	*ACD*	0.38506, 0.08643, 0.08132; 0.94149	0.42950	$\frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}] < c < \frac{4}{3}\sqrt{2}$
2.8	5/3/*h*9	*ADF*	0.40475, 0.20238, 0.07778; 1.14060	0.38572	$\frac{2}{7}\sqrt{6} < c < \frac{4}{3}\sqrt{2}$
*n*2.1	$h[6^3]^3$	*CDE*	{0.45, 0.1275, 0; 0.5}	>0.44621	$1-\frac{1}{3}\sqrt{3} < c < \frac{2}{7}\sqrt{6}$

P*$\bar{3}$*m*1 12*j **$0 \le y$; $2x-1 < y < \frac{1}{2}x$; $0 \le z \le \frac{1}{4}$**

A	$x-y$, $-y$, $-z$	*C*	x, y, $z+1$	*D*	$-x+y+1$, y, z	*F*	$-y$, $-x$, z
B	x, $x-y$, z		x, y, $z-1$	*E*	$x-y$, $-y$, $-z+1$		
0.1	6/3/*h*28	*ABCDE*	$\frac{4}{9}$, $\frac{1}{9}$, $\frac{1}{4}$, $\frac{2}{9}$			0.35828	
0.2	5/4/*h*5	*ABDEF*	$\frac{1}{3}$, 0, $\frac{1}{4}$, $\frac{2}{3}$			0.40307	
1.1	5/4/*h*17	*ABCD*	$\frac{1}{6}+\frac{1}{6}\sqrt{3}$, $\frac{1}{6}\sqrt{3}-\frac{1}{6}$, 0; $\frac{1}{2}-\frac{1}{6}\sqrt{3}$			0.32400	$\frac{1}{2}-\frac{1}{6}\sqrt{3} \le c < \frac{2}{9}$
1.2	4/4/*h*5	*ABDE*	$\frac{5}{12}$, $\frac{1}{12}$, $\frac{1}{4}$; $\frac{1}{6}\sqrt{6}$			0.27768	$\frac{2}{9} < c < \frac{2}{3}$

P*$\bar{3}$*c*1 12*i **0, $2x-1 \le y \le \frac{1}{2}x$; $0 \le z \le \frac{1}{4}$**

A	$x-y$, x, $-z$	*E*	x, $x-y$, $z+\frac{1}{2}$	*G*	$-x+y+1$, y, $z+\frac{1}{2}$	*I*	$-y$, $x-y$, z		
	y, $-x+y$, $-z$		x, $x-y$, $z-\frac{1}{2}$		$-x+y+1$, y, $z-\frac{1}{2}$		$-x+y$, $-x$, z		
B	$x-y$, $-y$, $-z+\frac{1}{2}$	*F*	$-y+1$, $x-y$, z	*H*	x, y, $z+1$	*J*	y, x, $-z+\frac{1}{2}$		
C	$x-y$, $-y$, $-z-\frac{1}{2}$		$-x+y+1$,$-x+1$, z		x, y, $z-1$	*K*	$-y$, $x-y-1$, z		
D	$-x+1$, $-y$, $-z$						$-x+y+1$, $-x$, z		
0.1	8/3/*h*15	*BCEGH*	$\frac{1}{6}+\frac{1}{6}\sqrt{3}$, $\frac{1}{6}\sqrt{3}-\frac{1}{6}$, 0; $\frac{1}{3}\sqrt{3}-\frac{1}{3}$			0.43201			
0.2	7/3/*h*14	*BCDEG*	$\frac{1}{6}+\frac{1}{6}\sqrt{3}$, $\frac{1}{6}\sqrt{3}-\frac{1}{6}$, 0; $1-\frac{1}{3}\sqrt{3}$			0.45821			
0.3	8/3/*h*12	*BDEFG*	$\frac{2}{9}\sqrt{10}-\frac{2}{9}$, $\frac{2}{9}\sqrt{10}-\frac{5}{9}$, $\frac{1}{16}$, $\frac{16}{9}\sqrt{2}-\frac{8}{9}\sqrt{5}$			0.46191			
0.4	7/3/*h*16	*ABCDE*	$\frac{1}{3}+\frac{1}{9}\sqrt{3}[2-(1+\sqrt{3})^{1/2}]$, $\frac{1}{9}[\sqrt{3}+(3-2\sqrt{3})(1+\sqrt{3})^{1/2}]$, 0; $\frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}]$				0.54567			
0.5	8/3/*h*16	*ABDEF*	0.42674, 0.14068, 0.04201; 0.70495			0.57044			
0.6	8/3/*h*11	*ABEFJ*	$\frac{2}{5}$, $\frac{1}{5}$, $\frac{1}{8}$; $\frac{4}{5}$			0.58042			
0.7	8/3/*h*10	*ABFIJ*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{8}$; $\frac{2}{3}\sqrt{6}$			0.55536			
0.8	10/3/*h*2	*ABDFIK*	$\frac{1}{3}$, 0, $\frac{1}{4}\sqrt{6}-\frac{1}{2}$; $\frac{2}{3}\sqrt{2}+\frac{2}{3}\sqrt{3}$			0.66568			
1.1	7/3/*h*18	*BEGH*	$\frac{7}{15}$, $\frac{2}{15}$, $\frac{1}{4}$, $\frac{2}{15}\sqrt{3}$			0.38694	$\frac{2}{15}\sqrt{3} \le c < \frac{1}{3}\sqrt{3}-\frac{1}{3}$		
1.2	6/4/*h*10	*BCEG*	$\frac{1}{6}+\frac{1}{6}\sqrt{3}$, $\frac{1}{6}\sqrt{3}-\frac{1}{6}$, 0; $\frac{1}{2}\sqrt{2}-\frac{1}{6}\sqrt{6}$			0.42089	$\frac{1}{3}\sqrt{3}-\frac{1}{3} < c < 1-\frac{1}{3}\sqrt{3}$		
1.3	6/3/*h*51	*BDEG*	0.46635, 0.13302, 0.03207; 0.46464			0.45071	$1-\frac{1}{3}\sqrt{3} < c < \frac{16}{9}\sqrt{2}-\frac{8}{9}\sqrt{5}$		
1.4	5/4/*h*18	*BCDE*	{0.43301, 0.11603, 0; 0.5}			>0.45821	$1-\frac{1}{3}\sqrt{3} < c < \frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}]$		
1.5	7/3/*h*19	*BEFG*	$\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}\sqrt{2}$			0.37024	$\frac{1}{3}\sqrt{2} \le c < \frac{16}{9}\sqrt{2}-\frac{8}{9}\sqrt{5}$		
1.6	7/3/*h*15	*DEFG*	$\frac{1}{3}\sqrt{6}-\frac{1}{3}$, $\frac{1}{3}\sqrt{6}-\frac{2}{3}$, 0; $2\sqrt{2}-\frac{4}{3}\sqrt{3}$			0.44882	$2\sqrt{2}-\frac{4}{3}\sqrt{3} \le c < \frac{16}{9}\sqrt{2}-\frac{8}{9}\sqrt{5}$		
1.7	6/3/*h*21	*BDFG*	$\frac{4}{7}$, $\frac{1}{7}$, $\frac{1}{8}$; $\frac{4}{7}$			0.29613	$\frac{16}{9}\sqrt{2}-\frac{8}{9}\sqrt{5} < c \le \frac{4}{7}$		
1.8	6/3/*h*52	*BDEF*	{0.45887, 0.14485, 0.05336; 0.6}			>0.46191	$\frac{16}{9}\sqrt{2}-\frac{8}{9}\sqrt{5} < c < 0.70495$		
1.9	5/4/*h*5	*ABCD*	$\frac{1}{3}$, 0, 0; $\frac{2}{3}$			0.40307	$\frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}] < c \le \frac{2}{3}$		
1.10	6/3/*h*53	*ABDE*	0.40246, 0.11030, 0.00446; 0.62195			0.54539	$\frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}] < c < 0.70495$		
1.11	7/3/*h*17	*ADEF*	$\frac{3}{7}$, $\frac{1}{7}$, 0; $\frac{2}{7}\sqrt{6}$			0.55975	$\frac{2}{7}\sqrt{6} \le c < 0.70495$		
1.12	7/3/*h*31	*ABEF*	0.41570, 0.16735, 0.08285; 0.74852			0.54424	$0.70495 < c < \frac{4}{5}$		
1.13	6/3/*h*54	*ABDF*	0.40429, 0.11270, 0.09234; 1.18996			0.46131	$0.70495 < c < \frac{2}{3}\sqrt{2}+\frac{2}{3}\sqrt{3}$		
1.14	6/3/*h*20	*BEFJ*	$1-\frac{1}{3}\sqrt{3}$, $\frac{1}{2}-\frac{1}{6}\sqrt{3}$, $\frac{1}{4}$; $\sqrt{3}-1$			0.48601	$\sqrt{3}-1 \le c < \frac{4}{5}$		

Table 1 (continued)

1.15	6/3/*h*22	*ABFJ*	$\frac{13}{12}-\frac{1}{12}\sqrt{73}$, $\frac{13}{24}-\frac{1}{24}\sqrt{73}$, $\frac{1}{8}$; $\frac{1}{3}(39-3\sqrt{73})^{1/2}$	0.51755	$\frac{4}{5} < c < \frac{2}{3}\sqrt{6}$
1.16	7/3/*h*32	*ABFI*	$\frac{1}{3}$, 0.12429, 0.11859; 1.73967	0.53825	$\frac{2}{3}\sqrt{6} < c < \frac{2}{3}\sqrt{2}+\frac{2}{3}\sqrt{3}$
1.17	6/3/*h*13	*BDFK*	$\frac{1}{2}$, 0, $\frac{1}{8}$, 2	0.45345	$2 \le c < \frac{2}{3}\sqrt{2}+\frac{2}{3}\sqrt{3}$
2.1	5/4/*h*21	*BEG*	0.48100, 0.14767, $\frac{1}{4}$; 0.35182	0.34503	$\frac{2}{15}\sqrt{3} < c < \frac{16}{9}\sqrt{2}-\frac{8}{9}\sqrt{5}$
2.2	5/4/*h*19	*DEG*	0.47385, 0.14051, 0; 0.48675	0.44621	$1-\frac{1}{3}\sqrt{3} < c < \frac{16}{9}\sqrt{2}-\frac{8}{9}\sqrt{5}$
2.3	4/6/*h*9	*BDE*	{0.45, 0.13332, 0.03502; 0.55}	>0.45071	$1-\frac{1}{3}\sqrt{3} < c < 0.70495$
2.4	5/3/*h*5	*BFG*	$\frac{1}{3}\sqrt{3}$, $\frac{2}{3}\sqrt{3}-1$, $\frac{1}{4}$; $4-2\sqrt{3}$	0.26045	$\frac{1}{3}\sqrt{2} < c < \frac{4}{7}$
2.5	5/3/*h*8	*BEF*	0.48786, 0.17112, $\frac{1}{4}$; 0.51627	0.36587	$\frac{1}{3}\sqrt{2} < c < \frac{4}{5}$
2.6	5/3/*h*5	*DFG*	$\frac{1}{3}\sqrt{3}$, $\frac{2}{3}\sqrt{3}-1$, 0; $4-2\sqrt{3}$	0.26045	$2\sqrt{2}-\frac{4}{3}\sqrt{3} < c < \frac{4}{7}$
2.7	5/3/*h*7	*DEF*	{0.47414, 0.14888, 0; 0.55}	>0.44882	$2\sqrt{2}-\frac{4}{3}\sqrt{3} < c < 0.70495$
2.8	4/3/*h*3	*BDF*	$\frac{1}{6}\sqrt{19}-\frac{1}{6}$, $\frac{1}{3}\sqrt{19}-\frac{4}{3}$, $\frac{1}{8}$; $\frac{2}{3}(6\sqrt{19}-24)^{1/2}$	0.24427	$\frac{16}{9}\sqrt{2}-\frac{8}{9}\sqrt{5} < c < \frac{2}{3}\sqrt{2}+\frac{2}{3}\sqrt{3}$
2.9	5/4/*h*20	*ADE*	0.40396, 0.11227, 0; 0.62688	0.54522	$\frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}] < c < 0.70495$
2.10	4/6/*h*2	*ABD*	$\frac{1}{3}$, 0, $\frac{1}{16}$; $\frac{2}{3}\sqrt{2}$	0.34009	$\frac{2}{3}[1-\sqrt{3}+(1+\sqrt{3})^{1/2}] < c < \frac{2}{3}\sqrt{2}+\frac{2}{3}\sqrt{3}$
2.11	6/3/*h*20	*AEF*	$1-\frac{1}{3}\sqrt{3}$, $\frac{1}{2}-\frac{1}{6}\sqrt{3}$, 0; $\sqrt{3}-1$	0.48601	$\frac{2}{7}\sqrt{6} < c < \frac{4}{5}$
2.12	5/3/*h*26	*ABF*	0.39875, 0.12304, 0.09712; 1.19533	0.45962	$0.70495 < c < \frac{2}{3}\sqrt{2}+\frac{2}{3}\sqrt{3}$
*n*3.1	$h[6^3]^3$	*DE*	{0.45, 0.1275, 0; 0.5}	>0.44621	$1-\frac{1}{3}\sqrt{3} < c < 0.70495$

R*$\bar{3}$*m* 36*i **$0 < x \le \frac{1}{3}$; $0 \le y < \frac{1}{2}x$; $0 \le z < \frac{1}{2}$**

A	$x-y, -y, -z$	*D*	$x, y, z+1$	*F*	$y+\frac{1}{3}, x-\frac{1}{3}, -z+\frac{2}{3}$	*I*	$-x+y+1, y, z$
B	$x, x-y, z$		$x, y, z-1$	*G*	$-x+\frac{2}{3}, -x+y+\frac{1}{3}, -z+\frac{4}{3}$		
C	$-x+\frac{2}{3}, -x+y+\frac{1}{3}, -z+\frac{1}{3}$	*E*	$x-y, -y, -z+1$	*H*	$-y, -x, z$		
0.1	6/3/*h*55	*BCDEG*	$\frac{2}{309}(11+3\sqrt{105})$, $\frac{1}{309}(9\sqrt{105}-70)$, $\frac{5}{12}$; $\frac{2}{103}(27-2\sqrt{105})$			0.34737	
0.2	6/3/*h*56	*ABCDE*	$\frac{4}{397}(36-\sqrt{105})$, $\frac{1}{397}(36-\sqrt{105})$, $\frac{1}{4}$; $\frac{2}{397}(36-\sqrt{105})$			0.36636	
0.3	5/4/*h*14	*BCEFG*	$\frac{1}{3}$, $\frac{16}{15}-\frac{2}{5}\sqrt{6}$, $\frac{5}{12}$; $\frac{8}{5}\sqrt{6}-\frac{18}{5}$			0.27718	
0.4	6/4/*h*9	*ABCEFH*	$\frac{9}{19}-\frac{2}{19}\sqrt{6}$, 0, $\frac{1}{4}$; $\frac{18}{19}-\frac{4}{19}\sqrt{6}$			0.50701	
0.5	5/4/*h*5	*ABCHI*	$\frac{1}{3}$, 0, $\frac{1}{12}$; 2			0.40307	
(0.5)		*BCFHI*	$\frac{1}{3}$, 0, $\frac{1}{4}$;				
(0.5)		*BEFHI*	$\frac{1}{3}$, 0, $\frac{5}{12}$;				
1.1	5/4/*h*40	*ABCD*	0.26408, 0.07055, 0.05625; 0.12298			0.32918	$0.12298 \le c < \frac{2}{397}(36-\sqrt{105})$
1.1′		*BDEG*					
1.2	5/4/*h*41	*BCDE*	0.26770, 0.07080, 0.38321; 0.12611			0.34616	$0.12611 \le c < \frac{2}{397}(36-\sqrt{105})$
1.3	4/4/*h*8	*BCEG*	0.29759, 0.07799, $\frac{5}{12}$; 0.25471			0.24263	$\frac{2}{105}(27-2\sqrt{105}) < c < \frac{8}{5}\sqrt{6}-\frac{18}{5}$
1.4	4/4/*h*10	*ABCE*	0.25201, 0.05309, $\frac{1}{4}$; 0.22635			0.29818	$\frac{2}{397}(36-\sqrt{105}) < c < \frac{18}{19}-\frac{4}{19}\sqrt{6}$
1.5	4/4/*h*9	*BCEF*	0.31001, 0.07172, 0.36430; 0.40882			0.24602	$\frac{8}{5}\sqrt{6}-\frac{18}{5} < c \le 0.47942$
1.6	4/4/*h*7	*ABCH*	0.22907, 0, 0.10319; 1.10993			0.23570	$\frac{18}{19}-\frac{4}{19}\sqrt{6} < c < 2$
1.6′		*BEFH*	0.22907, 0, 0.39681;				
1.7	4/4/*c*1	*BCFH*	$\frac{1}{4}$, 0, $\frac{1}{4}$; $\frac{1}{2}\sqrt{6}$			0.27768	$\frac{18}{19}-\frac{4}{19}\sqrt{6} < c < 2$
2.1	3/4/*c*1	*ABC*	$\frac{1}{6}+\frac{1}{12}\sqrt{2}$, $\frac{1}{6}\sqrt{2}-\frac{1}{6}$, $\frac{1}{6}\sqrt{2}-\frac{1}{6}$; $\frac{1}{4}\sqrt{6}$			0.11163	$0.12298 < c < 2$
2.1′		*BEG*					
(2.1)		*BCF*					
2.2	3/8/*h*1	*BEF*	$\frac{1}{3}$, $\frac{19}{24}-\frac{1}{8}\sqrt{33}$, $\frac{5}{12}$; $\frac{9}{4}-\frac{1}{4}\sqrt{33}$			0.17248	$\frac{8}{5}\sqrt{6}-\frac{18}{5} < c < 2$
*i*2.1	$h[3/4/c1]^2$	*BCE*	$\frac{1}{6}+\frac{1}{12}\sqrt{2}$, $\frac{1}{6}\sqrt{2}-\frac{1}{6}$, $\frac{5}{6}-\frac{1}{3}\sqrt{2}$; $\frac{1}{8}\sqrt{6}$			0.22327	$0.12611 < c < 0.47942$

R*$\bar{3}$*c* 36*f **$0 \le x \le \frac{1}{3}$; $0 \le y \le \frac{1}{2}x$; $0 \le z < \frac{1}{2}$**

A	$x-y, x, -z$	*G*	$x, y, z+1$	*L*	$x-y, x, -z+1$	*Q*	$-y+1, x-y, z$
	$y, -x+y, -z$		$x, y, z-1$		$y, -x+y, -z+1$		$-x+y+1, -x+1, z$
B	$x-y, -y, -z+\frac{1}{2}$	*H*	$-y, x-y, z$	*M*	$x-y+\frac{1}{3}, -y+\frac{2}{3}, -z+\frac{1}{6}$	*R*	$x-y-\frac{1}{3}, x-\frac{2}{3}, -z+\frac{1}{3}$
C	$x-y, -y, -z-\frac{1}{2}$		$-x+y, -x, z$	*N*	$x-y+\frac{1}{3}, x-\frac{1}{3}, -z+\frac{2}{3}$		$y+\frac{2}{3}, -x+y+\frac{1}{3}, -z+\frac{1}{3}$
D	$-x+\frac{2}{3}, -x+y+\frac{1}{3}, -z-\frac{1}{6}$	*I*	$-x+\frac{2}{3}, -y+\frac{1}{3}, -z+\frac{1}{3}$		$y+\frac{1}{3}, -x+y+\frac{2}{3}, -z+\frac{2}{3}$	*S*	$-y, x-y-1, z$
E	$y+\frac{1}{3}, x-\frac{1}{3}, -z+\frac{1}{6}$	*J*	$-x+\frac{2}{3}, -x+y+\frac{1}{3}, -z+\frac{5}{6}$	*O*	$-x+1, -y, -z$		$-x+y+1, -x, z$
F	$x, x-y, z+\frac{1}{2}$	*K*	$y, x, -z+\frac{1}{2}$	*P*	$-x+\frac{1}{3}, -y-\frac{1}{3}, -z+\frac{2}{3}$	*T*	$-x+1, -y, -z+1$
	$x, x-y, z-\frac{1}{2}$						
0.1	7/3/*h*33	*BDFGJ*	$\frac{1}{11586}(1243-1161\sqrt{3}+486\sqrt{35}+96\sqrt{105})$, $\frac{1}{11586}(-1120-1890\sqrt{3}+387\sqrt{35}+291\sqrt{105})$, $\frac{1}{6}$; $\frac{1}{1931}(873+387\sqrt{3}-162\sqrt{35}-32\sqrt{105})$			0.38553	
0.2	7/3/*h*34	*BCDFG*	$\frac{1}{5638}(981+630\sqrt{3}-93\sqrt{35}-8\sqrt{105})$, $\frac{1}{5638}(72+279\sqrt{3}-162\sqrt{35}+77\sqrt{105})$, 0; $\frac{1}{2819}(72+279\sqrt{3}-162\sqrt{35}+77\sqrt{105})$			0.40778	
0.3	6/3/*h*59	*BDFIJ*	$\frac{1}{78}(413+225\sqrt{3}-96\sqrt{17}-54\sqrt{51})$, $\frac{1}{78}(544+306\sqrt{3}-129\sqrt{17}-75\sqrt{51})$, $\frac{1}{6}$; $\frac{1}{13}(-225-129\sqrt{3}+54\sqrt{17}+32\sqrt{51})$			0.34157	
0.4	5/4/*h*14	*BDEIJ*	$\frac{1}{3}$, $\frac{16}{15}-\frac{2}{5}\sqrt{6}$, $\frac{1}{6}$; $-\frac{18}{5}+\frac{8}{5}\sqrt{6}$			0.27718	
0.5	7/3/*h*35	*ABCDF*	0.22280, 0.05970, 0; 0.34182			0.50752	
0.6	7/3/*h*5	*ABDFI*	0.23139, 0.08184, 0.05648; 0.39341			0.49811	
0.7	6/3/*h*1	*BFIJK*	$\frac{140}{339}-\frac{2}{113}\sqrt{105}$, $\frac{70}{339}-\frac{1}{113}\sqrt{105}$, $\frac{7}{24}$; $-\frac{36}{113}+\frac{8}{113}\sqrt{105}$			0.45038	
0.8	7/3/*h*1	*ABFIK*	$\frac{36}{73}-\frac{2}{73}\sqrt{105}$, $\frac{18}{73}-\frac{1}{73}\sqrt{105}$, $\frac{1}{8}$; $\frac{72}{73}-\frac{4}{73}\sqrt{105}$			0.49102	

Table 1 (continued)

0.9	7/3/*h*1	*BFJKL*	$\frac{36}{73}-\frac{2}{73}\sqrt{105}$, $\frac{18}{73}-\frac{1}{73}\sqrt{105}$, $\frac{3}{8}$; $\frac{72}{73}-\frac{4}{73}\sqrt{105}$	0.49102	
0.10	6/4/*h*9	*ABCDE*	$\frac{9}{19}-\frac{2}{19}\sqrt{6}$, 0, 0; $\frac{18}{19}-\frac{4}{19}\sqrt{6}$	0.50701	
0.11	6/4/*h*11	*ABDEI*	0.24634, 0.05146, 0.06875; 0.61617	0.49128	
0.12	7/3/*h*2	*ABHIK*	$\frac{2}{11}$, $\frac{1}{11}$, $\frac{1}{8}$; $\frac{4}{11}\sqrt{6}$	0.49569	
0.13	7/3/*h*2	*BHJKL*	$\frac{2}{11}$, $\frac{1}{11}$, $\frac{3}{8}$; $\frac{4}{11}\sqrt{6}$	0.49569	
0.14	7/3/*h*36	*ABEHI*	0.18926, 0.02851, 0.11319; 1.10409	0.56547	
0.15	6/4/*h*1	*ADEIM*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{24}$; $\frac{1}{2}\sqrt{6}$	0.51013	
(0.15)		*BIJKN*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{7}{24}$;		
0.16	7/3/*h*3	*AEHIM*	0.24417, 0.12209, 0.07897; 1.89350	0.56478	
0.17	7/3/*h*3	*BHJKN*	0.24417, 0.12209, 0.32897; 1.89350	0.56478	
0.18	7/3/*h*37	*BHJNP*	0.23820, 0.04760, 0.33670; 2.12831	0.55304	
0.19	8/3/*c*2	*AEHMQ*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{24}$; $2\sqrt{6}$	0.55536	
(0.19)		*BHKNQ*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{7}{24}$;		
0.20	7/3/*h*4	*EHIMQ*	$\frac{1}{3}$, $\frac{1}{6}$, $-\frac{1}{12}+\frac{1}{12}\sqrt{6}$; $3+\sqrt{6}$	0.49926	
(0.20)		*BHIKQ*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{5}{12}-\frac{1}{12}\sqrt{6}$;		
0.21	10/3/*h*2	*AEHORS*	$\frac{1}{3}$, 0, $-\frac{1}{6}+\frac{1}{12}\sqrt{6}$; $2\sqrt{2}+2\sqrt{3}$	0.66568	
(0.21)		*EHIQRS*	$\frac{1}{3}$, 0, $\frac{1}{3}-\frac{1}{12}\sqrt{6}$;		
(0.21)		*HJNPQS*	$\frac{1}{3}$, 0, $\frac{1}{6}+\frac{1}{12}\sqrt{6}$;		
(0.21)		*HJLQST*	$\frac{1}{3}$, 0, $\frac{2}{3}-\frac{1}{12}\sqrt{6}$;		
0.21′		*BHIQRS*	$\frac{1}{3}$, 0, $\frac{1}{12}\sqrt{6}$;		
(0.21′)		*BHNPQS*	$\frac{1}{3}$, 0, $\frac{1}{2}-\frac{1}{12}\sqrt{6}$;		
1.1	6/3/*h*57	*BFGJ*	0.26048, 0.07422, 0.30625; 0.12937	0.36431	$0.12937 \le c < \frac{1}{2819}(72+279\sqrt{3}-162\sqrt{35}+77\sqrt{105})$
1.2	6/3/*h*58	*BDFG*	0.26420, 0.07458, 0.13321; 0.13285	0.38412	$0.13285 \le c < \frac{1}{2819}(72+279\sqrt{3}-162\sqrt{35}+77\sqrt{105})$
1.3	5/4/*h*42	*BDFJ*	0.27207, 0.08298, $\frac{1}{6}$; 0.20568	0.34142	$\frac{1}{1931}(873+387\sqrt{3}-162\sqrt{35}-32\sqrt{105})$ $< c < \frac{1}{13}(-225-129\sqrt{3}+54\sqrt{17}+32\sqrt{51})$
1.4	5/4/*h*43	*BCDF*	0.24934, 0.06681, 0; 0.18702	0.38327	$\frac{1}{2819}(72+279\sqrt{3}-162\sqrt{35}+77\sqrt{105}) < c < 0.34182$
1.5	4/4/*h*14	*BDIJ*	0.30946, 0.08234, $\frac{1}{6}$; 0.28954	0.25657	$\frac{1}{13}(-225-129\sqrt{3}+54\sqrt{17}+32\sqrt{51}) < c < -\frac{18}{5}+\frac{8}{5}\sqrt{6}$
1.6	5/3/*h*27	*BDFI*	0.26858, 0.08239, 0.14419; 0.22524	0.33894	$\frac{1}{13}(-225-129\sqrt{3}+54\sqrt{17}+32\sqrt{51}) < c < 0.39341$
1.7	5/3/*h*28	*BFIJ*	0.26106, 0.08945, 0.24282; 0.26284	0.30829	$\frac{1}{13}(-225-129\sqrt{3}+54\sqrt{17}+32\sqrt{51})$ $< c < -\frac{36}{113}+\frac{8}{113}\sqrt{105}$
1.8	4/4/*h*15	*BDEI*	0.31932, 0.07916, 0.12973; 0.38362	0.25611	$-\frac{18}{5}+\frac{8}{5}\sqrt{6} < c < 0.61617$
1.9	6/3/*h*60	*ABDF*	0.22773, 0.07281, 0.03570; 0.37172	0.49121	$0.34182 < c < 0.39341$
1.10	6/3/*h*61	*BFJL*	0.21852, 0.08376, 0.42707; 0.38477	0.44599	$0.34182 < c < \frac{72}{73}-\frac{4}{73}\sqrt{105}$
1.11	5/4/*h*44	*ABCD*	0.22055, 0.03249, 0; 0.39681	0.48114	$0.34182 < c < \frac{18}{19}-\frac{4}{19}\sqrt{6}$
1.12	5/4/*h*1	*BFIK*	0.22360 0.11180, 0.22188; 0.38976	0.41330	$0.38976 \le c < \frac{72}{73}-\frac{4}{73}\sqrt{105}$
1.13	6/3/*h*1	*ADFI*	$\frac{140}{339}-\frac{2}{113}\sqrt{105}$, $\frac{70}{339}-\frac{1}{113}\sqrt{105}$, $\frac{1}{24}$; $-\frac{36}{113}+\frac{8}{113}\sqrt{105}$	0.45038	$0.39341 < c \le -\frac{36}{113}+\frac{8}{113}\sqrt{105}$
1.14	6/3/*h*62	*ABFI*	0.22053, 0.09412, 0.09451; 0.40814	0.47033	$0.39341 < c < \frac{72}{73}-\frac{4}{73}\sqrt{105}$
1.15	5/4/*h*45	*ABDI*	0.23788, 0.06685, 0.06290; 0.50553	0.46979	$0.39341 < c < 0.61617$
1.16	5/4/*h*2	*BFJK*	0.22776, 0.11388, 0.30840; 0.40572	0.44786	$0.40572 \le c < \frac{72}{73}-\frac{4}{73}\sqrt{105}$
1.17	4/6/*h*4	*BIJK*	0.25523, 0.12762, $\frac{7}{24}$; 0.74163	0.35482	$-\frac{36}{113}+\frac{8}{113}\sqrt{105} < c < \frac{1}{2}\sqrt{6}$
1.18	5/4/*h*3	*ABIK*	0.20168, 0.10084, $\frac{1}{8}$; 0.62201	0.44750	$\frac{72}{73}-\frac{4}{73}\sqrt{105} < c < \frac{4}{11}\sqrt{6}$
1.19	5/4/*h*3	*BJKL*	0.20168, 0.10084, $\frac{3}{8}$; 0.31100	0.44750	$\frac{72}{73}-\frac{4}{73}\sqrt{105} < c < \frac{4}{11}\sqrt{6}$
1.20	5/4/*h*46	*ABDE*	0.23010, 0.02373, 0.04332; 0.53270	0.45926	$\frac{18}{19}-\frac{4}{19}\sqrt{6} < c < 0.61617$
1.21	5/4/*h*47	*ABEI*	0.23281, 0.04525, 0.08981; 0.74978	0.46834	$0.61617 < c < 1.10409$
1.22	5/4/*h*48	*ADEI*	0.27656, 0.08834, 0.04758; 0.91842	0.41555	$0.61617 < c < \frac{1}{2}\sqrt{6}$
1.23	6/3/*h*63	*ABHI*	0.18301, 0.07709, 0.12069; 0.93241	0.48887	$\frac{4}{11}\sqrt{6} < c < 1.10409$
1.24	6/3/*h*64	*BHJL*	0.17866, 0.05300, 0.38397; 0.96858	0.46878	$\frac{4}{11}\sqrt{6} < c < 1.10409$
(1.24)		*ABEH*			
1.25	5/3/*c*3	*BHIK*	$\frac{2}{3}-\frac{1}{3}\sqrt{2}$, $\frac{1}{3}-\frac{1}{6}\sqrt{2}$, $\frac{1}{12}+\frac{1}{12}\sqrt{2}$; $\sqrt{6}$	0.22327	$\frac{4}{11}\sqrt{6} < c < 3+\sqrt{6}$
1.26	6/3/*h*65	*AEHI*	0.21061, 0.06402, 0.08914; 1.48352	0.49859	$1.10409 < c < 1.89350$
1.27	5/3/*h*29	*BHJP*	0.18851, 0.00256, 0.35200; 1.58968	0.46710	$1.10409 < c < 2.12831$
(1.27)		*BEHI*			
1.28	5/4/*h*4	*AEIM*	0.30806, 0.15403, 0.05925; 1.46851	0.47896	$\frac{1}{2}\sqrt{6} < c < 1.89350$
1.29	5/4/*h*4	*BJKN*	0.30806, 0.15403, 0.30925; 1.46851	0.47896	$\frac{1}{2}\sqrt{6} < c < 1.89350$
1.30	6/3/*h*66	*BHJN*	0.24205, 0.07971, 0.33441; 2.03381	0.54237	$1.89350 < c < 2.12831$
1.31	6/3/*h*2	*AEHM*	0.26526, 0.13263, 0.05222; 3.11064	0.44076	$1.89350 < c < 2\sqrt{6}$
1.32	6/3/*h*2	*BHKN*	0.26526, 0.13263, 0.30222; 3.11064	0.44076	$1.89350 < c < 2\sqrt{6}$
1.33	5/3/*h*1	*EHIM*	0.26686, 0.13343, 0.11198; 3.50555	0.39824	$1.89350 < c < 3+\sqrt{6}$
1.34	5/4/*h*5	*BJNP*	$\frac{1}{3}$, 0, $\frac{1}{3}$; 2	0.40307	$2 \le c < 2.12831$
(1.34)		*ADEO*	$\frac{1}{3}$, 0, 0;		
(1.34)		*BEIR*	$\frac{1}{3}$, 0, $\frac{1}{6}$;		
1.35	6/3/*h*67	*BHNP*	0.25305, 0.04436, 0.30828; 3.41471	0.42462	$2.12831 < c < 2\sqrt{2}+2\sqrt{3}$
1.36	6/3/*h*68	*HJNP*	0.25638, 0.04348, 0.36063; 3.47502	0.43680	$2.12831 < c < 2\sqrt{2}+2\sqrt{3}$
1.37	7/3/*h*38	*AEHQ*	$\frac{1}{3}$, 0.12429, 0.03953; 5.21901	0.53825	$2\sqrt{6} < c < 2\sqrt{2}+2\sqrt{3}$
1.37′		*BHNQ*	$\frac{1}{3}$, 0.12429, 0.29380;		
1.38	7/3/*h*4	*HJNQ*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{6}+\frac{1}{12}\sqrt{6}$; $3+\sqrt{6}$	0.49926	$3+\sqrt{6} \le c < 2\sqrt{2}+2\sqrt{3}$

Table 1 (continued)

(1.38)		*HJLQ*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{3}-\frac{1}{12}\sqrt{6}$;		
1.39	6/3/*h*69	*BHIQ*	$\frac{1}{3}$, 0.13512, 0.21071; 5.66532	0.48887	$3+\sqrt{6} < c < 2\sqrt{2}+2\sqrt{3}$
1.39′		*EHIQ*	$\frac{1}{3}$, 0.13512, 0.12263;		
*i*1.1	$c[5/3/c3]^2$	*BHJK*	$\frac{2}{3}-\frac{1}{3}\sqrt{2}$, $\frac{1}{3}-\frac{1}{6}\sqrt{2}$, $\frac{7}{12}-\frac{1}{6}\sqrt{2}$; $\frac{1}{2}\sqrt{6}$	0.44653	$\frac{4}{11}\sqrt{6} < c < 1.89350$
2.1	4/4/*h*11	*BFJ*	0.25439, 0.08276, 0.30732; 0.23109	0.29174	$0.12937 < c < \frac{72}{73}-\frac{4}{73}\sqrt{105}$
2.2	4/4/*h*12	*BDF*	0.26472, 0.08019, 0.12705; 0.21057	0.33666	$0.13285 < c < 0.39341$
2.3	4/4/*h*13	*BFI*	0.25644, 0.08979, 0.21928; 0.27247	0.30572	$\frac{1}{13}(-225-129\sqrt{3}+54\sqrt{17}+32\sqrt{51}) < c < \frac{72}{73}-\frac{4}{73}\sqrt{105}$
2.4	3/10/*h*4	*BDI*	0.30551, 0.07898, 0.13739; 0.33721	0.24725	$\frac{1}{13}(-225-129\sqrt{3}+54\sqrt{17}+32\sqrt{51}) < c < 0.61617$
2.5	3/8/*h*3	*BIJ*	0.30722, 0.09102, 0.25646; 0.47174	0.18120	$\frac{1}{13}(-225-129\sqrt{3}+54\sqrt{17}+32\sqrt{51}) < c < \frac{1}{2}\sqrt{6}$
2.5′		*DEI*			
2.6	3/8/*h*1	*BEI*	$\frac{1}{3}$, $\frac{19}{24}-\frac{1}{8}\sqrt{33}$, $\frac{1}{6}$; $\frac{9}{4}-\frac{1}{4}\sqrt{33}$	0.17248	$-\frac{18}{5}+\frac{8}{5}\sqrt{6} < c < 2.12831$
2.6′		*BJP*			
2.7	5/4/*h*1	*FJL*	0.22360 0.11180, 0.47188; 0.38976	0.41330	$0.34182 < c < \frac{72}{73}-\frac{4}{73}\sqrt{105}$
2.7′		*ADF*			
2.8	4/6/*c*3	*BJL*	$\frac{5}{24}$, $\frac{1}{24}$, $\frac{5}{12}$; $\frac{1}{4}\sqrt{6}$	0.36072	$0.34182 < c < 1.10409$
(2.8)		*ABE*			
2.9	4/6/*h*10	*ABD*	0.22962, 0.04184, 0.03971; 0.48193	0.45044	$0.34182 < c < 0.61617$
2.10	3/6/*h*1	*BIK*	0.23974, 0.11987, 0.21748; 1.66238	0.16799	$0.38976 < c < 3+\sqrt{6}$
(2.10)		*EIM*			
2.11	5/4/*h*2	*AFI*	0.22776, 0.11388, 0.05840; 0.40572	0.44786	$0.39341 < c < \frac{72}{73}-\frac{4}{73}\sqrt{105}$
2.12	4/6/*h*11	*ABI*	0.21492, 0.07744, 0.10267; 0.62683	0.41303	$0.39341 < c < 1.10409$
2.13	4/6/*h*4	*ADI*	0.25523, 0.12762, $\frac{1}{24}$; 0.74163	0.35482	$0.39341 < c < \frac{1}{2}\sqrt{6}$
2.14	4/4/*c*1	*ADE*	$\frac{1}{4}$, 0, 0; $\frac{1}{2}\sqrt{6}$	0.27768	$\frac{18}{19}-\frac{4}{19}\sqrt{6} < c < 2.12831$
(2.14)		*BJN*			
2.15	4/6/*h*12	*AEI*	0.26240, 0.08268, 0.06386; 1.06568	0.39887	$0.61617 < c < 1.89350$
2.16	4/3/*h*8	*BHI*	0.19435, 0.07528, 0.19851; 2.55862	0.21613	$\frac{4}{11}\sqrt{6} < c < 2\sqrt{2}+2\sqrt{3}$
(2.16)		*BHP*			
2.17	5/3/*c*3	*HJL*	$\frac{2}{3}-\frac{1}{3}\sqrt{2}$, $\frac{1}{3}-\frac{1}{6}\sqrt{2}$, $\frac{1}{3}+\frac{1}{12}\sqrt{2}$; $\sqrt{6}$	0.22327	$\frac{4}{11}\sqrt{6} < c < 2\sqrt{2}+2\sqrt{3}$
(2.17)		*AEH*			
2.18	4/3/*h*7	*EHI*	0.22490, 0.04368, 0.12260; 3.08244	0.32337	$1.10409 < c < 2\sqrt{2}+2\sqrt{3}$
(2.18)		*HJP*			
2.19	4/6/*c*2	*BKN*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{7}{24}$; $\sqrt{6}$	0.39270	$\frac{1}{2}\sqrt{6} < c < 2\sqrt{6}$
(2.19)		*AEM*	$\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{24}$;		
2.20	5/3/*h*30	*BHN*	0.25999, 0.07918, 0.30610; 3.34684	0.41559	$1.89350 < c < 2\sqrt{2}+2\sqrt{3}$
2.21	5/3/*h*1	*HJN*	0.26686, 0.13343, 0.36198; 3.50555	0.39824	$1.89350 < c < 2\sqrt{2}+2\sqrt{3}$
2.22	4/6/*h*2	*JNP*	$\frac{1}{3}$, 0, $\frac{17}{48}$; $2\sqrt{2}$	0.34009	$2 < c < 2\sqrt{2}+2\sqrt{3}$
(2.22)		*AEO*	$\frac{1}{3}$, 0, $\frac{1}{48}$;		
(2.22)		*EIR*	$\frac{1}{3}$, 0, $\frac{7}{48}$;		
(2.22)		*JLT*	$\frac{1}{3}$, 0, $\frac{23}{48}$;		
2.22′		*BNP*	$\frac{1}{3}$, 0, $\frac{5}{16}$;		
(2.22′)		*BIR*	$\frac{1}{3}$, 0, $\frac{3}{16}$;		
*i*2.1	$h[3/6/h1]^2$	*BJK*	0.23974, 0.11987, 0.31506; 0.83119	0.33598	$0.40572 < c < 1.89350$
*i*2.2	$c[5/3/c3]^2$	*AHI*	$\frac{2}{3}-\frac{1}{3}\sqrt{2}$, $\frac{1}{3}-\frac{1}{6}\sqrt{2}$, $\frac{1}{3}-\frac{1}{6}\sqrt{2}$; $\frac{1}{2}\sqrt{6}$	0.44653	$\frac{4}{11}\sqrt{6} < c < 1.89350$
*i*2.3	$h[4/3/h5]^2{}_{II}$	*BHJ*	0.19127, 0.05734, 0.35525; 1.31706	0.42202	$\frac{4}{11}\sqrt{6} < c < 2.12831$
(*i*2.3)		*BEH*			
3.1	3/6/*h*1	*JL*	0.23974, 0.11987, 0.46747; 1.66238	0.16799	$0.34182 < c < 2\sqrt{2}+2\sqrt{3}$
(3.1)		*AD*			
(3.1)		*AE*			
(3.1)		*BN*			
(3.1)		*JN*			
*i*3.1	$h[3/6/h1]^2$	*AI*	0.23974, 0.11987, 0.06506; 0.83119	0.33598	$0.39341 < c < 1.89350$

or metrical parameters – then one capital letter symbolizes the whole set.

(iii) A list of all corresponding types of sphere packings and of interpenetrating sphere packings and layers forms the third block.

The dimension d of the parameter range under consideration and a serial number j are combined to a symbol $d.j$ in the first column. A letter i or n precedes this symbol in the case of interpenetrating sphere packings or layers, respectively.

The type of sphere configuration is identified in the second column. Each sphere-packing type is designated by its symbol $k/m/fi$, as was first introduced by Fischer (1971): k means the number of contacts per sphere, m is the length of the shortest ring of spheres with mutual contact within the sphere packing, f indicates the highest crystal family for a sphere packing of that type (h: hexagonal, c: cubic), and i is an arbitrary number. The symbols for types of interpenetrating sphere packings have the form $f_1[k/m/fi]^l$. Here, l is the number of sphere

Table 2
Layer and rod descriptions for the sphere packings of the 147 trigonal types.

Type	Symmetry	Layer description			Rod description		
3/6/*h*1	$R\bar{3}m$ 18*h*	–			–		
3/6/*h*2	$P\bar{3}1c$ 12*i*	–			–		
3/8/*h*1	$R\bar{3}m$ 18*f*	–			–		
3/8/*h*3	$R\bar{3}c$ 36*f*	–			–		
3/10/*h*4	$R\bar{3}c$ 36*f*	–			–		
4/3/*h*1	$R32$ 9*d*	3.12^2	1,0 2	−−	–		
4/3/*h*2	$R\bar{3}c$ 18*e*	3.12^2	1,0 3	−−	–		
4/3/*h*5	$R32$ 18*f*	3.12^2	1,0 3	−+	–		
4/3/*h*6	$P\bar{3}1c$ 12*i*	3.12^2	1,0 2	−+			
4/3/*h*7	$R\bar{3}c$ 36*f*	3.12^2	1,0 6	−+	–		
		3.12^2	1,0 6	−+			
4/3/*h*8	$R\bar{3}c$ 36*f*	3.12^2	1,0 6	−+	–		
4/4/*h*6	$P\bar{3}1c$ 12*i*	–			$6^3(6,3)$	1,0	p
4/4/*h*7	$R\bar{3}m$ 36*i*	4.6.12	1,0 3	−+	–		
4/4/*h*8	$R\bar{3}m$ 36*i*	–			$6^3(3,2)$	1,0	g_h
4/4/*h*9	$R\bar{3}m$ 36*i*	–			$4^4(0,2)$	1,0	g_h
4/4/*h*10	$R\bar{3}m$ 36*i*	–			$6^3(6,3)$	1,0	p_h
					$6^3(3,2)$	1,0	g_h
4/4/*h*11	$R\bar{3}c$ 36*f*	–			$6^3(6,3)$	1,0	p_h
4/4/*h*12	$R\bar{3}c$ 36*f*	–			$6^3(6,3)$	1,0	p_h
4/4/*h*13	$R\bar{3}c$ 36*f*	–			$6^3(6,3)$	1,0	p_h
4/4/*h*14	$R\bar{3}c$ 36*f*	–			$6^3(3,2)$	1,0	g_h
4/4/*h*15	$R\bar{3}c$ 36*f*	–			$4^4(0,2)$	1,0	g_h
4/5/*h*1	$R\bar{3}c$ 18*e*	–			$6^3(6,3)$	1,0	p_h
4/5/*h*2	$R\bar{3}c$ 18*e*	–			–		
4/6/*h*4	$R\bar{3}m$ 18*h*	–			$6^3(3,2)$	1,0	g_h
4/6/*h*5	$R\bar{3}$ 18*f*	6^3	1,0 3	−+	–		
4/6/*h*6	$P3_221$ 6*c*	–			–		
4/6/*h*7	$R3c$ 18*b*	–			–		
4/6/*h*8	$P\bar{3}1c$ 12*i*	6^3	1,0 3	−+	$6^3(0,3)$	1,0	p
4/6/*h*9	$P\bar{3}c1$ 12*g*	–			$6^3(6,3)$	1,0	p
4/6/*h*10	$R\bar{3}c$ 36*f*	–			$6^3(0,3)$	1,0	p_h
4/6/*h*11	$R\bar{3}c$ 36*f*	–			$6^3(0,3)$	1,0	p_h
4/6/*h*12	$R\bar{3}c$ 36*f*	–			–		
5/3/*h*1	$R\bar{3}m$ 18*h*	3464	1,0 3	++	–		
		3.12^2	2,0 3	−+			
5/3/*h*4	$P3_221$ 6*c*	–			–		
5/3/*h*21	$R\bar{3}$ 18*f*	3.12^2	1,1 3	−+	–		
5/3/*h*22	$R32$ 18*f*	3.12^2	$1,1^+$ 3	−+	$4^4(0,3)$	1,0	p_h
					$6^3(3,2)$	1,1	p_h'
5/3/*h*23	$R32$ 18*f*	3.12^2	1,1 3	++	$4^4(0,2)$	1,1	p_h'
5/3/*h*24	$P\bar{3}1c$ 12*i*	–			$6^3(6,3)$	2,0	p
5/3/*h*25	$P\bar{3}1c$ 12*i*	3.12^2	2,0 2	−+	–		
5/3/*h*26	$P\bar{3}c1$ 12*g*	3464	1,0 2	−+	$6^3(0,3)$	1,1	p
		3.12^2	2,0 2	−+			
5/3/*h*27	$R\bar{3}c$ 36*f*	–			$6^3(6,3)$	2,0	p_h
5/3/*h*28	$R\bar{3}c$ 36*f*	–			$6^3(6,3)$	2,0	p_h
5/3/*h*29	$R\bar{3}c$ 36*f*	3.12^2	1,1 6	−+	–		
		3.12^2	1,1 6	−+			
5/3/*h*30	$R\bar{3}c$ 36*f*	3464	1,0 6	++	–		
5/4/*h*1	$R\bar{3}m$ 18*h*	–			$4^4(0,6)$	1,0	p_h
					$4^4(1,6)$	1,0	g_h
5/4/*h*2	$R\bar{3}m$ 18*h*	–			$4^4(0,6)$	q	p_h
					$4^4(2,6)$	q	g_h
5/4/*h*3	$R\bar{3}m$ 18*h*	–			$4^4(3,3)$	1,0	p_h
					$6^3(3,2)$	d	g_h
5/4/*h*4	$R\bar{3}m$ 18*h*	–			$4^4(0,2)$	d	g_h
5/4/*h*8	$P3_221$ 6*c*	–			$6^3(3,2)$	1,1	p
					$6^3(3,2)$	c	g
5/4/*h*9	$P3_221$ 6*c*	–			–		
5/4/*h*10	$P3_221$ 6*c*	–			$6^3(3,2)$	1,1	p
5/4/*h*12	$P3_221$ 6*c*	6^3	2,0 3	−+	–		

Table 2 (continued)

Type	Symmetry	Layer description			Rod description		
5/4/*h*14	$R\bar{3}m$ 18*f*	–			$4^4(0,12)$	q	p_h
					$4^4(1,3)$	1,0	g_h
5/4/*h*38	$R\bar{3}$ 18*f*	–			–		
5/4/*h*39	$R\bar{3}$ 18*f*	6^3	2,0 3	−+	–		
5/4/*h*40	$R\bar{3}m$ 36*i*	–			$4^4(0,12)$	1,0	p_h
					$4^4(1,6)$	1,0	g_h
5/4/*h*41	$R\bar{3}m$ 36*i*	–			$4^4(0,12)$	1,0	p_h
					$4^4(2,6)$	1,0	g_h
5/4/*h*42	$R\bar{3}c$ 36*f*	–			$6^3(6,3)$	2,0	p_h
					$6^3(3,2)$	2,0	g_h
5/4/*h*43	$R\bar{3}c$ 36*f*	–			$4^4(6,6)$	1,0	p_h
					$6^3(3,2)$	2,0	g_h
5/4/*h*44	$R\bar{3}c$ 36*f*	–			$4^4(0,6)$	1,0	p_h
					$6^3(3,2)$	2,0	g_h
5/4/*h*45	$R\bar{3}c$ 36*f*	–			$6^3(0,3)$	2,0	p_h
5/4/*h*46	$R\bar{3}c$ 36*f*	–			$6^3(0,3)$	1,1	p_h
					$4^4(0,2)$	1,1	g_h
5/4/*h*47	$R\bar{3}c$ 36*f*	–			$6^3(0,3)$	1,1	p_h
5/4/*h*48	$R\bar{3}c$ 36*f*	–			–		
6/3/*h*1	$R\bar{3}m$ 18*h*	–			$4^4(0,6)$	2,0	p_h
					$3^34^2(3,2)$	1,0	g_h
6/3/*h*2	$R\bar{3}m$ 18*h*	3464	2,0 3	++	–		
6/3/*h*8	$P3_221$ 6*c*	–			–		
6/3/*h*9	$P3_221$ 6*c*	–			–		
6/3/*h*11	$P3_221$ 6*c*	6^3	2,1 3	++	–		
6/3/*h*14	$R3m$ 9*b*	–			–		
6/3/*h*15	$R\bar{3}c$ 18*e*	–			$3^34^2(6,3)$	1,0	p_h
					$4^4(1,3)$	2,0	g_h
6/3/*h*16	$R\bar{3}c$ 18*e*				$6^3(0,3)$	2,1	p_h
6/3/*h*18	$R32$ 9*d*	3636	1,1 3	++	–		
6/3/*h*19	$R\bar{3}c$ 18*e*	3.12^2	2,1 3	−−	–		
6/3/*h*23	$R3c$ 18*b*	–			$4^4(0,3)$	1,1	p_h
6/3/*h*24	$R3c$ 18*b*	–			–		
6/3/*h*25	$R3c$ 18*b*	3.12^2	$1,1,1^+$ 3	−−	$4^4(0,3)$	1,1	p_h
6/3/*h*44	$R\bar{3}$ 18*f*	3^46	1,0 3	−+	–		
		3.12^2	3,0 3	−+			
6/3/*h*45	$R\bar{3}$ 18*f*	–			–		
6/3/*h*46	$R\bar{3}$ 18*f*	–			–		
6/3/*h*47	$R\bar{3}$ 18*f*	6^3	2,1 3	−+	–		
6/3/*h*48	$R32$ 18*f*	3636	1,1 6	++	–		
6/3/*h*49	$P\bar{3}1c$ 12*i*	6^3	2,1 2	−+	$3^2434(0,3)$	1,0	p
6/3/*h*50	$P\bar{3}1c$ 12*i*	3^46	1,0 2	−+	$6^3(0,3)$	2,1	p
6/3/*h*51	$P\bar{3}c1$ 12*g*	–			$6^3(6,3)$	3,0	p
					$4^4(3,3)$	2,0	g
6/3/*h*52	$P\bar{3}c1$ 12*g*	3.12^2	2,1 2	−+	$6^3(6,3)$	2,1	p
		3.12^2	2,1 2	−+	$4^4(0,3)$	2,0	g
6/3/*h*53	$P\bar{3}c1$ 12*g*	6^3	2,1 2	−+	$3^2434(0,3)$	1,0	p
6/3/*h*54	$P\bar{3}c1$ 12*g*	3^46	1,0 2	−+	$6^3(0,3)$	2,1	p
		3.12^2	3,0 2	−+			
6/3/*h*55	$R\bar{3}m$ 36*i*	–			$4^4(0,12)$	2,0	p_h
					$3^34^2(3,2)$	1,0	g_h
6/3/*h*56	$R\bar{3}m$ 36*i*	–			$3^34^2(6,3)$	1,0	p_h
					$3^34^2(3,2)$	1,0	g_h
6/3/*h*57	$R\bar{3}c$ 36*f*	–			$3^34^2(6,3)$	1,0	p_h
					$4^4(1,6)$	2,0	g_h
6/3/*h*58	$R\bar{3}c$ 36*f*	–			$3^34^2(6,3)$	1,0	p_h
					$4^4(2,6)$	2,0	g_h
6/3/*h*59	$R\bar{3}c$ 36*f*	–			$6^3(6,3)$	3,0	p_h
					$6^3(3,2)$	3,0	g_h
6/3/*h*60	$R\bar{3}c$ 36*f*	–			$3^2434(0,3)$	1,0	p_h
6/3/*h*61	$R\bar{3}c$ 36*f*	–			$3^2434(0,3)$	1,0	p_h
6/3/*h*62	$R\bar{3}c$ 36*f*	–			$3^2434(0,3)$	1,0	p_h
6/3/*h*63	$R\bar{3}c$ 36*f*	3.12^2	2^+,1 6	−+	$3^34^2(0,3)$	1,0	p_h
6/3/*h*64	$R\bar{3}c$ 36*f*	3.12^2	$2,1^+$ 6	−+	$3^34^2(0,3)$	1,0	p_h

Table 2 (continued)

Type	Symmetry	Layer description			Rod description		
6/3/*h*65	$R\bar{3}c$ 36*f*	3.12^2	2,1 6	++	–		
		3.12^2	3,0 6	−+			
6/3/*h*66	$R\bar{3}c$ 36*f*	3464	1,1 6	++	–		
		3.12^2	2,1 6	−+			
6/3/*h*67	$R\bar{3}c$ 36*f*	$3^4$6	1,0 6	++	–		
6/3/*h*68	$R\bar{3}c$ 36*f*	$3^4$6	1,0 6	−+	–		
		3.12^2	3,0 6	−+			
6/3/*h*69	$R\bar{3}c$ 36*f*	3636	1,1 12	++	–		
6/4/*h*1	$R\bar{3}m$ 9*e*	–			4^4(0,6)	*c*	p_h
					4^4(1,3)	*c*	g_h
6/4/*h*4	$P3_221$ 6*c*	6^3	2,1 3	−+	–		
6/4/*h*7	*R*32 9*d*	3.12^2	$1,1,1^+$ 1.5	−−	4^4(0,3)	1,1	p_h
					4^4(1,3)	1,1	p_h'
					4^4(1,6)	*c*	p_h''
6/4/*h*8	$R\bar{3}c$ 18*e*	–			4^4(3,3)	1,1	p_h
6/4/*h*9	$R\bar{3}m$ 18*f*	–			4^4(0,6)	1,1	p_h
					4^4(1,3)	1,1	g_h
6/4/*h*11	$R\bar{3}c$ 36*f*	–			6^3(0,3)	2,1	p_h
					4^4(0,2)	2,1	g_h
7/3/*h*1	$R\bar{3}m$ 18*h*	–			3^6(3,6)	1,0	p_h
					3^34^2(3,2)	*q*	g_h
7/3/*h*2	$R\bar{3}m$ 18*h*	3.12^2	2,2 3	−+	3^6(0,3)	1,0	p_h
					6^3(3,2)	*d*	g_h
7/3/*h*3	$R\bar{3}m$ 18*h*	3464	2,1 3	−+	4^4(0,2)	*d*	g_h
7/3/*h*4	$R\bar{3}m$ 18*h*	3636	2,1 6	++	–		
7/3/*h*5	$R\bar{3}c$ 36*f*	–			$3^2$434(0,3)	2,0	p_h
7/3/*h*9	$P3_221$ 6*c*	–			6^3(3,2)	2,2	*p*
7/3/*h*10	$P3_221$ 6*c*	–			–		
7/3/*h*11	$P3_221$ 6*c*	6^3	3,1 3	−+	–		
7/3/*h*23	$R\bar{3}$ 18*f*	3.12^2	3,1 3	−+	–		
7/3/*h*24	$R\bar{3}$ 18*f*	3.12^2	3,1 3	−+	–		
7/3/*h*25	$R\bar{3}$ 18*f*	–			4^4(0,6)	2,1	p_h
					4^4(1,3)	2,1	g_h
7/3/*h*26	$R\bar{3}$ 18*f*	–			4^4(3,3)	2,1	p_h
7/3/*h*27	$R\bar{3}$ 18*f*	–			–		
7/3/*h*28	$R\bar{3}$ 18*f*	$3^4$6	2,0 3	−+	–		
7/3/*h*29	$R\bar{3}$ 18*f*	6^3	3,1 3	−+	–		
7/3/*h*30	$P3_221$ 6*c*	–			3^34^2(3,2)	1,1	*p*
					4^4(1,3)	2,1	*g*
7/3/*h*31	$P\bar{3}c1$ 12*g*	3464	2,1 2	−+	$3^2$434(0,3)	1,1	*p*
		3.12^2	3,1 2	−+	4^4(0,3)	2,1	*g*
7/3/*h*32	$P\bar{3}c1$ 12*g*	3636	2,1 4	++	3^34^2(0,3)	1,1	*p*
7/3/*h*33	$R\bar{3}c$ 36*f*	–			3^34^2(6,3)	2,0	p_h
					3^34^2(3,2)	2,0	g_h
7/3/*h*34	$R\bar{3}c$ 36*f*	–			3^6(6,12)	1,0	p_h
					3^34^2(3,2)	2,0	g_h
7/3/*h*35	$R\bar{3}c$ 36*f*	–			3^6(0,6)	1,0	p_h
					6^3(3,2)	3,1	g_h
7/3/*h*36	$R\bar{3}c$ 36*f*	3.12^2	2^+,1,1 6	−+	3^34^2(0,3)	1,1	p_h
		3.12^2	2,1,1^+ 6	−+			
7/3/*h*37	$R\bar{3}c$ 36*f*	$3^4$6	1,1 6	−+	–		
		3.12^2	$3,1^+$,0 6	−+			
7/3/*h*38	$R\bar{3}c$ 36*f*	3636	2,1 12	++	–		
7/4/*h*1	$R\bar{3}$ 18*f*	–			4^4(0,6)	2,1	p_h
					4^4(1,3)	2,1	g_h
8/3/*h*1	$R\bar{3}$ 18*f*	–			4^4(0,6)	3,1	p_h
					4^4(1,3)	3,1	g_h
8/3/*h*2	$R\bar{3}c$ 18*e*	–			$3^2$434(0,3)	*c*	p_h
8/3/*h*5	$P3_221$ 6*c*	6^3	3,2 3	−+	–		
8/3/*h*7	*R*3*m* 9*b*	–			4^4(0,3)	2,1,1	p_h
					4^4(1,3)	*c*	g_h
8/3/*h*8	$R\bar{3}c$ 18*e*	–			3^6(3,6)	1,1	p_h
					4^4(1,3)	2,2	g_h

Table 2 (continued)

Type	Symmetry	Layer description			Rod description		
8/3/*h*9	$R\bar{3}c$ 18*e*	–			3^6(0,3)	1,1	p_h
8/3/*h*12	$P\bar{3}c1$ 12*g*	3.12^2	3,2 2	−+	6^3(6,3)	4,1	*p*
		3.12^2	3,2 2	−+	3^6(0,3)	2,0	*g*
8/3/*h*16	$P\bar{3}c1$ 12*g*	$3^4$6	2,1 2	−+	$3^2$434(0,3)	2,1	*p*
		3.12^2	4,1 2	−+	4^4(0,3)	3,1	*g*
9/3/*h*1	$R\bar{3}$ 18*f*	–			3^6(3,6)	2,1	p_h
					4^4(1,3)	3,2	g_h
9/3/*h*2	$R\bar{3}$ 18*f*	3.12^2	$3,2^+$,1 3	−+	3^6(0,3)	2,1	p_h
10/3/*h*1	$R\bar{3}$ 18*f*	$3^4$6	3,2 3	++	–		
10/3/*h*4	$R\bar{3}m$ 6*c*	3^6	3,1 6	++	–		
11/3/*h*2	$P3_221$ 6*c*	3^6	3,2 6	++	–		

packings that interpenetrate each other, *k*/*m*/*fi* describes their type, and f_1 identifies the highest crystal family where interpenetrating sphere packings of this type occur. *f* and f_1 may differ in principle (*cf. e.g.* $h[3/4/c1]^2$ in $R\bar{3}m$ 36*i*). Interpenetrating sphere layers are designated in a similar way: instead of the symbol of the sphere-packing type, one of the usual symbols for the Shubnikov nets (Shubnikov, 1916) are given; *l* means the number of sets of parallel sphere layers, *i.e.* 3 in the present case.

A string of capital letters in the third column identifies all neighbouring points that give rise to sphere contacts. The next two columns refer to those special sphere packings (or interpenetrating sphere packings or layers) of the type under consideration that show minimal density: in the fourth column, the respective coordinate parameters *x*, *y* and *z* and the axial ratio *c*/*a* are shown, in the fifth column the value ρ_m of the minimal density. If the parameters are included in braces, the corresponding type of sphere packing does not comprise an arrangement with minimal density within its parameter region. Then the given parameters refer to an arbitrarily chosen sphere packing of that type. For each type of sphere configuration with at least one free parameter, the range of the axial ratio *c*/*a* is shown in the sixth column.

For some space groups, the asymmetric unit of the Euclidean normalizer is not totally bounded by mirror planes ($R\bar{3}$, *R*32, $R\bar{3}m$, $R\bar{3}c$ with normalizer $R\bar{3}m$ and $P3_212$, $P3_221$ with normalizer $P6_422$). Then, the following situation may occur for some types of sphere configuration: one can either choose a connected parameter region (identified by a certain string of capital letters) that is not completely located within the asymmetric unit under consideration, or one may choose a disconnected parameter region that belongs completely to this asymmetric unit. In the latter case, each part of the parameter region is represented by another string of capital letters. The corresponding sets of symmetry operations, however, can be transformed into one another by a symmetry operation belonging either to the space group itself or to its Euclidean normalizer. The different parts of such a disconnected parameter range are marked by adding parentheses or a prime, respectively, in the first column of Table 1. Fischer (1991) has discussed in detail a corresponding tetragonal example.

3. Discussion

In total, sphere packings of 225 types can be realized in lattice complexes with trigonal characteristic space groups. Enhanced cubic symmetry may occur for 13, enhanced hexagonal symmetry for 65 of these types. Trigonal inherent symmetry occurs for all sphere packings of 147 types. For 6 of these types, however, the sphere packings can be generated only in trigonal lattice complexes with less than three degrees of freedom (4/6/*h*7, 6/3/*h*14, 6/3/*h*23, 6/3/*h*24, 6/3/*h*25, 8/3/*h*7). In addition to the types of sphere packing with three contacts per sphere derived by Koch & Fischer (1995), one further such type, namely 3/10/*h*4, is found in $R\bar{3}c$.

All interpenetrating sphere packings with trigonal symmetry belong to 7 types (*cf.* also Koch *et al.*, 2006). There is only one way to fit two individual packings of type 3/4/*c*1, 3/6/*h*1, 3/10/*h*1, 4/3/*h*1 or 5/3/*c*3 into each other thus forming interpenetrating sphere packings of types $h[3/4/c1]^2$, $h[3/6/h1]^2$, $h[3/10/h1]^2$, $h[4/3/h1]^2$ or $c[5/3/c3]^2$, respectively. There are two different ways, however, to intertwine two packings of type 4/3/*h*5 with individual symmetry *R*32 (*cf.* Fig. 1): the combination of two such packings with the same handedness results in interpenetrating sphere packings of type $h[4/3/h5]^2{}_{\rm I}$ with symmetry *R*32 again, whereas the combination of two enantiomorphic packings leads to type $h[4/3/h5]^2{}_{\rm II}$ with symmetry $R\bar{3}c$. All interpenetrating sphere packings that belong to the types $h[3/4/c1]^2$, $h[3/6/h1]^2$, $h[4/3/h1]^2$, $h[4/3/h5]^2{}_{\rm I}$ and $h[4/3/h5]^2{}_{\rm II}$ show trigonal inherent symmetry. In contrast to that, the inherent symmetry may become cubic for interpenetrating sphere packings with specialized metric belonging to type $c[5/3/c3]^2$. A pair of sphere packings that interpenetrate each other and belong to type $h[3/10/h1]^2$ can show either trigonal symmetry, namely $P3_221$, or hexagonal symmetry, namely $P6_122$. No such pair, however, is compatible with both symmetries (*cf.* below and Koch *et al.*, 2006). Interpenetration of more than two sphere packings is impossible with trigonal symmetry.

Three sets of interpenetrating 6^3 layers of spheres can be formed in $P\bar{3}1c$ 12*i* as well as in $P\bar{3}c1$ 12*g*. As the inherent symmetry is *P*6/*mcc* – *m*.. for some of these sets, the type $h[6^3]^3$ has already been described before (Sowa & Koch, 2004).

Table 2 gives a survey of all those sphere-packing types where the inherent symmetry is trigonal for each packing. [Similar information on the hexagonal types is tabulated in a previous paper (Sowa & Koch, 2005)]. In the first column, the sphere-packing type is identified by its symbol. The second column shows the lattice complex where packings of that type can be generated with highest site symmetry. Most of the trigonal sphere packings can be subdivided either into connected layer-like or into connected rod-like subunits, or into both of these. Such subunits may be used to construct some kind of 'descriptive symbols' that reflect certain properties of the sphere packings but are not sufficient to discriminate all types. The next columns give information on such layer-like and rod-like subunits, respectively, and on their mutual arrangement.

(i) Connected layer-like arrangements of spheres perpendicular to **c** are found for 61 of the 147 trigonal types. These flat or corrugated sphere layers correspond directly to the vertex-transitive plane nets (Shubnikov, 1916) and are characterized in column 3 by the well known symbols 3^6, 6^3, 3.12^2, $3^4 6$, 3636, 3464 and 4.6.12. Column 4 shows the numbers of contacts between a certain sphere and other spheres from the layers above and below[1] and, in addition, the number of (possibly corrugated) sphere layers per translation period. A similar symbolism has been used before (Koch & Fischer, 1999; Sowa & Koch, 2005). The first sign in column 5 shows whether or not a sphere packing with plane layers belongs to the regarded type. The second sign is + if there is only one possibility to subdivide the sphere packing into the corrugated layers under consideration, otherwise the sign is −.

(ii) The sphere packings of 86 trigonal types may be subdivided into connected rod-like arrangements of spheres running parallel to **c**. Such a rod can be regarded as part of a plane net that is rolled up. In order to identify uniquely a type of rod-like arrangement of spheres, the symbol of the net (column 6) is supplemented by the vector between the centres of two arbitrary spheres that coincide when the net is rolled up (*cf.* Koch & Fischer, 1978; Sowa & Koch, 2005). Each sphere is in contact with other spheres from one or two neighbouring rods.[2] The respective numbers are given in column 7. In a few cases, the congruent rods of spheres are not disjunct but have part of their spheres in common, namely rows of dumb-bells, chains or quadrangular ribbons of spheres, designated by *d*, *c* or *q*, respectively. The last column describes the position of the rod axes with respect to a hexagonal unit cell (*p*: rods around 00*z*; *g*: $\frac{21}{33}z$ and $\frac{12}{33}z$; p_h: 00*z*, $\frac{21}{33}z$ and $\frac{12}{33}z$; p'_h: $\frac{1}{3}0z$, $0\frac{1}{3}z$ and $\frac{22}{33}z$; p''_h: $\frac{2}{3}0z$, $0\frac{2}{3}z$ and $\frac{11}{33}z$; g_h: $\frac{1}{3}0z$, $0\frac{1}{3}z$, $\frac{22}{33}z$, $\frac{2}{3}0z$, $0\frac{2}{3}z$ and $\frac{11}{33}z$).

Sphere packings of 21 trigonal types cannot be subdivided either into connected layer-like or into connected rod-like subunits.

Altogether, sphere packings of 330 types refer to the hexagonal crystal family. For 13 of these types, the inherent symmetry is cubic for some of their sphere packings. 105 types exist where each generating space group belongs to the hexagonal crystal system. Sphere packings of 65 types are compatible with hexagonal as well as with trigonal symmetry.

Types 4/6/*h*14, 5/4/*h*6 and $h[3/10/h1]^2$ show an unusual property that is worth noting: all three types may be generated with site symmetry 1 in $P6_122$ as well as in $P3_221$, but not with higher site symmetry in a common supergroup. The reason for this behaviour is the following: both regarded lattice complexes have a common limiting complex, *i.e.* ^+Q with characteristic Wyckoff position $P6_222$ 3*c* 222; the point configurations referring to this limiting complex, however, do not belong to the interior of the two parameter regions under

[1] Sphere packings of a few trigonal types split up into corrugated 3.12^2 layers in such a way that at one side each sphere is connected to the next but one corrugated layer instead of the next layer. This is indicated by the superscript + at the respective number of sphere contacts (*cf.* 5/3/*h*22, 6/3/*h*63, 6/3/*h*64). In other cases, sphere contacts exist to the next and to the next but one layer on the same side (6/3/*h*25, 6/4/*h*7, 7/3/*h*36, 7/3/*h*37, 9/3/*h*2).

[2] The only exception is sphere-packing type 8/3/*h*7 where each sphere has contact to three neighbouring rods.

Table 3
Minimal densities and sample parameters for all hexagonal sphere-packing types.

Type	Symmetry	df_{min}	df_{max}	$x, y, z; a, c$	ρ_{min}
3/4/*h*1*a*	$P6_222$ 12*k*	2		0.45337, 0.12740, 0.07476; 3.86895, 3.48257	0.13918
3/4/*h*1*b*	$P6_222$ 12*k*	2		0.43223, 0.18515, 0.38400; 2.52646, 2.52646	>0.39382
3/4/*h*2*a*	$P6_222$ 12*k*	2		0.43559, 0.09867, 0.09899; 3.51466, 4.03834	0.14544
3/4/*h*2*b*	$P6_222$ 12*k*	2		0.43126, 0.16211, 0.36000; 2.43538, 2.60586	>0.39382
3/4/*h*3	$P6_222$ 12*k*	2		0.53120, 0.14951, 0.10710; 3.66002, 4.42491	0.12240
3/6/*h*1	$R\bar{3}m$ 18*h*	2	3	0.11987, 0.23974, 0.43494; 4.27156, 3.55047	0.16799
3/6/*h*2	$P\bar{3}1c$ 12*i*	2		0.44883, 0.11550, 0.09283; 3.03007, 2.38989	0.33065
3/6/*h*3	$P6_322$ 12*i*	2		0.44096, 0.10763, 0.14714; 3.88012, 2.34650	0.20537
3/8/*h*1	$R\bar{3}m$ 18*f*	1	3	0.40693, 0.00000, 0.00000; 5.37228, 2.18614	0.17248
3/8/*h*2	$P6_222$ 6*i*	1	3	0.40693, 0.81386, 0.00000; 3.10169, 2.18614	0.17248
3/8/*h*3	$R\bar{3}c$ 36*f*	2		0.30722, 0.09102, 0.25646; 6.33826, 2.98998	0.18120
3/8/*h*4	$P6_122$ 12*c*	2		0.48832, 0.11851, 0.10345; 3.95351, 2.82448	0.16434
3/8/*h*5	$P6_222$ 12*k*	2		0.42483, 0.19254, 0.38409; 2.54207, 2.28786	>0.46383
3/8/*h*6	$P6_222$ 12*k*	2		0.37633, 0.14769, 0.27430; 2.12480, 4.57637	0.35115
3/10/*h*1	$P6_222$ 6*f*	1	3	0.50000, 0.00000, 0.12500; 1.88562, 4.00000	0.25507
3/10/*h*2	$P6_122$ 12*c*	2		0.50282, 0.14350, 0.45567; 3.95721, 2.03740	0.22740
3/10/*h*3	$P6_122$ 12*c*	2		0.42689, 0.11006, 0.27909; 3.88677, 1.51998	0.31596
3/10/*h*4	$R\bar{3}c$ 36*f*	2		0.30551, 0.07898, 0.13739; 6.39108, 2.15515	0.24725
3/12/*h*1	$P6_222$ 6*g*	1	3	0.29796, 0.00000, 0.00000; 2.47481, 2.02635	0.29229
4/3/*h*1	$R32$ 9*d*	1	2	0.20000, 0.00000, 0.00000; 2.88675, 2.23607	0.29202
4/3/*h*2	$R\bar{3}c$ 18*e*	1	2	0.21922, 0.00000, 0.25000; 2.63361, 3.77442	0.41571
4/3/*h*3	$P6_3/mmc$ 6*h*	1	2	0.44018, 0.88036, 0.25000; 3.11963, 1.52596	0.24427
4/3/*h*4	$P6_3/mmc$ 12*k*	1	2	0.42982, 0.85964, 0.08796; 3.45463, 3.08570	0.19701
4/3/*h*5	$R32$ 18*f*	2		0.19127, 0.05734, 0.10525; 3.39581, 4.47250	0.21101
4/3/*h*6	$P\bar{3}1c$ 12*i*	2		0.47503, 0.14749, 0.04000; 3.05787, 1.98761	>0.33776
4/3/*h*7	$R\bar{3}c$ 36*f*	2		0.22490, 0.04368, 0.12260; 2.79508, 8.61565	0.32337
4/3/*h*8	$R\bar{3}c$ 36*f*	2		0.19435, 0.07528, 0.19851; 3.40161, 8.70345	0.21613
4/3/*h*9*a*	$P6_222$ 12*k*	1		0.45817, 0.11429, 0.09211; 3.57190, 3.83831	0.14815
4/3/*h*9*b*	$P6_222$ 12*k*	1		0.43522, 0.17698, 0.39788; 2.30672, 3.46228	0.39382
4/3/*h*10	$P6_322$ 12*i*	2		0.47152, 0.14355, 0.11109; 2.99894, 2.99894	>0.26274
4/4/*h*1	$P6_222$ 6*g*	0	2	0.25000, 0.00000, 0.00000; 2.00000, 2.59808	0.34907
4/4/*h*2	$P6_222$ 6*i*	1	2	0.21132, 0.42264, 0.00000; 2.23071, 1.73205	0.42089
4/4/*h*3	$P6_222$ 6*i*	1	3	0.19282, 0.38564, 0.00000; 1.85942, 2.35143	0.44621
4/4/*h*4	$P6_222$ 6*i*	1	3	0.27598, 0.55196, 0.00000; 1.96837, 2.82269	0.33170
4/4/*h*5	$P6_3/mmc$ 12*j*	1	2	0.41667, 0.08333, 0.25000; 4.00000, 1.63299	0.27768
4/4/*h*6	$P\bar{3}1c$ 12*i*	2		0.39501, 0.06497, 0.10000; 3.12100, 1.87260	>0.38052
4/4/*h*7	$R\bar{3}m$ 36*i*	1		0.22907, 0.00000, 0.10319; 4.36556, 4.84547	0.23570
4/4/*h*8	$R\bar{3}m$ 36*i*	1		0.29759, 0.07799, 0.41667; 7.06199, 1.79875	0.24263
4/4/*h*9	$R\bar{3}m$ 36*i*	1		0.31001, 0.07172, 0.36430; 6.00373, 2.45442	0.24602
4/4/*h*10	$R\bar{3}m$ 36*i*	1		0.25201, 0.05309, 0.25000; 6.85758, 1.55220	0.29818
4/4/*h*11	$R\bar{3}c$ 36*f*	2		0.25439, 0.08276, 0.30732; 6.86004, 1.58531	0.29174
4/4/*h*12	$R\bar{3}c$ 36*f*	2		0.26472, 0.08019, 0.12705; 6.74622, 1.42056	0.33666
4/4/*h*13	$R\bar{3}c$ 36*f*	2		0.25644, 0.08979, 0.21928; 6.39305, 1.74191	0.30572
4/4/*h*14	$R\bar{3}c$ 36*f*	1		0.30946, 0.08234, 0.16667; 6.64172, 1.92307	0.25657
4/4/*h*15	$R\bar{3}c$ 36*f*	1		0.31932, 0.07916, 0.12973; 6.05087, 2.32121	0.25611
4/4/*h*16	$P6_222$ 12*k*	1		0.55518, 0.16296, 0.29327; 3.48287, 2.37769	0.25155
4/4/*h*17	$P6_222$ 12*k*	1		0.53657, 0.14477, 0.25000; 3.83460, 1.92307	0.25657
4/4/*h*18	$P6_222$ 12*k*	1		0.53061, 0.26531, 0.07959; 1.98885, 5.74176	0.31945
4/4/*h*19	$P6_222$ 12*k*	1		0.39798, 0.19899, 0.26136; 1.88571, 5.27998	>0.37959
4/4/*h*20	$P6_222$ 12*k*	1		0.26269, 0.02538, 0.08726; 1.99231, 5.70795	0.32022
4/4/*h*21	$P6_222$ 12*k*	1		0.31767, 0.08512, 0.39266; 2.11273, 4.42629	0.36722
4/4/*h*22	$P6_222$ 12*k*	1		0.33228, 0.02560, 0.29669; 2.74914, 2.44092	0.39328
4/4/*h*23	$P6_222$ 12*k*	1		0.42259, 0.08038, 0.25000; 3.65875, 1.72110	0.31490
4/4/*h*24	$P6_222$ 12*k*	1		0.41580, 0.18577, 0.37326; 2.46314, 2.40574	0.49708
4/4/*h*25	$P6_222$ 12*k*	1		0.44811, 0.14176, 0.41667; 3.93939, 1.52262	0.30704
4/4/*h*26	$P6_222$ 12*k*	1		0.41422, 0.17988, 0.40033; 2.12925, 3.75397	0.42629
4/4/*h*27	$P6_222$ 12*k*	1		0.42537, 0.16069, 0.35516; 2.40055, 2.56859	>0.46396
4/4/*h*28	$P6_222$ 12*k*	1		0.43941, 0.16555, 0.36026; 2.50857, 2.48563	0.46383
4/4/*h*29	$P6_222$ 12*k*	1		0.41446, 0.18078, 0.27671; 2.12317, 4.51506	0.35647
4/4/*h*30	$P6_122$ 12*c*	1		0.51719, 0.14160, 0.00000; 4.07724, 1.79875	0.24263
4/4/*h*31	$P6_122$ 12*c*	1		0.42193, 0.09963, 0.25000; 3.90971, 1.47620	0.32152
4/4/*h*32	$P6_122$ 12*c*	1		0.55516, 0.16255, 0.45700; 3.47685, 2.37534	0.25267
4/4/*h*33	$P6_122$ 12*c*	2		0.33172, 0.00000, 0.17067; 2.63078, 2.92965	0.35782

Table 3 (continued)

Type	Symmetry	df_{min}	df_{max}	$x, y, z; a, c$	ρ_{min}
4/4/*h*34	$P6_122$ 12*c*	2		0.34926, 0.07370, 0.20305; 2.50827, 3.60169	0.32018
4/4/*h*35	$P6_122$ 12*c*	2		0.40749, 0.16922, 0.33400; 2.66369, 1.97113	>0.50123
4/4/*h*36	$P6_122$ 12*c*	2		0.34922, 0.12575, 0.22199; 2.27094, 4.30995	0.32641
4/4/*h*37	$P6_222$ 12*k*	1		0.35098, 0.10020, 0.33524; 2.30202, 2.78208	0.49211
4/4/*h*38	$P6_222$ 12*k*	1		0.31767, 0.08512, 0.27401; 2.11273, 4.42629	0.36722
4/4/*h*39	$P6_222$ 12*k*	1		0.29053, 0.08105, 0.26254; 1.92554, 5.05331	0.38723
4/4/*h*40	$P6_222$ 12*k*	1		0.29167, 0.08333, 0.40476; 1.92154, 5.04404	>0.38733
4/4/*h*41	*P*6/*mcc* 24*m*	1		0.45534, 0.12201, 0.18750; 4.46142, 2.66667	0.27338
4/4/*h*42	*P*6/*mcc* 24*m*	1		0.47227, 0.13894, 0.14062; 3.23254, 3.55579	>0.37959
4/4/*h*43	*P*6/*mcc* 24*m*	1		0.43150, 0.11562, 0.14016; 3.10200, 3.56730	>0.37959
4/4/*h*44	$P6_3/mcm$ 24*l*	1		0.46429, 0.13096, 0.07778; 4.40868, 2.90324	0.25715
4/4/*h*45	$P6_3/mmc$ 24*l*	1		0.43760, 0.10427, 0.09522; 4.36556, 3.23031	0.23570
4/5/*h*1	$R\bar{3}c$ 18*e*	1	2	0.43053, 0.00000, 0.25000; 5.30213, 1.35249	0.28622
4/5/*h*2	$R\bar{3}c$ 18*e*	1	2	0.26212, 0.00000, 0.25000; 3.05962, 2.77815	0.41845
4/5/*h*3	$P6_122$ 6*b*	1	2	0.23648, 0.47297, 0.25000; 2.26132, 2.26160	0.31367
4/5/*h*4	$P6_122$ 6*b*	1	2	0.43053, 0.86106, 0.25000; 3.06119, 1.35249	0.28622
4/6/*h*1	$P6_222$ 3*c*	0	3	0.50000, 0.00000, 0.00000; 1.63299, 1.73205	0.39270
4/6/*h*2	$P6_3/mmc$ 4*f*	1	2	0.33333, 0.66667, 0.06250; 1.63299, 2.66667	0.34009
4/6/*h*3	$P6_122$ 6*a*	1	2	0.38285, 0.00000, 0.00000; 2.51554, 1.61566	0.35482
4/6/*h*4	$R\bar{3}m$ 18*h*	1	2	0.12762, 0.25523, 0.08333; 4.35704, 1.61566	0.35482
4/6/*h*5	$R\bar{3}$ 18*f*	2		0.28910, 0.03128, 0.27441; 2.61313, 3.62299	0.43990
4/6/*h*6	$P3_221$ 6*c*	2		0.39518, 0.08000, 0.42521; 1.74711, 2.00917	>0.57681
4/6/*h*7	*R*3*c* 18*b*	2		0.23500, 0.08896, 0.00000; 3.02528, 1.96995	>0.51013
4/6/*h*8	$P\bar{3}1c$ 12*i*	2		0.38506, 0.08643, 0.08132; 2.61793, 2.46476	0.42950
4/6/*h*9	$P\bar{3}c1$ 12*g*	2		0.45000, 0.13332, 0.03502; 3.02551, 1.66403	>0.45071
4/6/*h*10	$R\bar{3}c$ 36*f*	2		0.22962, 0.04184, 0.03971; 4.64567, 2.23890	0.45044
4/6/*h*11	$R\bar{3}c$ 36*f*	2		0.21492, 0.07744, 0.10267; 4.38072, 2.74596	0.41303
4/6/*h*12	$R\bar{3}c$ 36*f*	2		0.26240, 0.08268, 0.06386; 3.71339, 3.95729	0.39887
4/6/*h*13	$P6_122$ 12*c*	1		0.44058, 0.13031, 0.33333; 3.89882, 1.42507	0.33492
4/6/*h*14	$P6_122$ 12*c*	1		0.44251, 0.05749, 0.37264; 1.88116, 3.85639	>0.51013
	$P3_221$ 6*c*	2		0.40000, 0.06086, 0.41667; 1.78568, 1.96425	
5/3/*h*1	$R\bar{3}m$ 18*h*	1	2	0.13343, 0.26686, 0.22397; 2.49817, 4.37873	0.39824
5/3/*h*2	$P6_222$ 12*k*	0		0.28868, 0.07735, 0.40377; 1.93185, 5.01910	0.38733
5/3/*h*3	$P6_122$ 12*c*	2		0.39470, 0.16940, 0.32635; 1.87504, 4.59486	0.44912
5/3/*h*4	$P3_221$ 6*c*	2		0.53614, 0.19783, 0.30502; 1.96159, 2.67581	0.35233
5/3/*h*5	*P*6/*mmm* 6*l*	0	2	0.42265, 0.84530, 0.00000; 3.73205, 1.00000	0.26045
5/3/*h*6	$P6_2$ 6*c*	1		0.42374, 0.18110, 0.00000; 2.18378, 1.78307	0.42661
5/3/*h*7	*P*6/*mcc* 12*l*	1	2	0.47414, 0.14888, 0.00000; 3.06087, 1.68348	>0.44882
5/3/*h*8	$P6_3/mcm$ 12*j*	1	2	0.48786, 0.17112, 0.25000; 3.37404, 1.74191	0.36587
5/3/*h*9	$P6_3/mmc$ 12*k*	1	2	0.20238, 0.40475, 0.07778; 2.54535, 2.90324	0.38572
5/3/*h*10	$P6_222$ 12*k*	0		0.44027, 0.16318, 0.35849; 2.50177, 2.49847	0.46396
5/3/*h*11	$P6_222$ 12*k*	0		0.42265, 0.21132, 0.06904; 1.93185, 5.12132	0.37959
5/3/*h*12	$P6_122$ 12*c*	1		0.33342, 0.00317, 0.17036; 2.62841, 2.93466	0.35785
5/3/*h*13	$P6_122$ 12*c*	1		0.41968, 0.17899, 0.36256; 2.54390, 2.23670	0.50123
5/3/*h*14	$P6_122$ 12*c*	1		0.36626, 0.13279, 0.20657; 2.34731, 3.94218	0.33402
5/3/*h*15	$P6_122$ 12*c*	1		0.29917, 0.20083, 0.42919; 1.98356, 5.11079	0.36081
5/3/*h*16	$P6_222$ 12*k*	0		0.28868, 0.07735, 0.26289; 1.93185, 5.01910	0.38733
5/3/*h*17	$P6_322$ 12*i*	1		0.48468, 0.15135, 0.10120; 3.17256, 2.74351	0.26274
5/3/*h*18	*P*6/*mcc* 24*m*	0		0.45534, 0.12201, 0.14645; 3.34607, 3.41421	0.37959
5/3/*h*19	*P*6/*mcc* 24*m*	1		0.45647, 0.14678, 0.13285; 2.89506, 3.76358	>0.40348
5/3/*h*20	$P6_3/mcm$ 24*l*	1		0.46055, 0.18432, 0.12230; 3.13227, 3.91534	>0.33284
5/3/*h*21	$R\bar{3}$ 18*f*	2		0.20301, 0.01824, 0.30000; 2.96790, 2.07753	>0.59132
5/3/*h*22	*R*32 18*f*	1		0.17988, 0.04565, 0.25000; 3.56483, 1.91892	0.44628
5/3/*h*23	*R*32 18*f*	1		0.19342, 0.03552, 0.29585; 3.23740, 2.40011	0.43263
5/3/*h*24	$P\bar{3}1c$ 12*i*	1		0.41667, 0.08333, 0.08856; 3.29150, 1.75988	0.38052
5/3/*h*25	$P\bar{3}1c$ 12*i*	1		0.46832, 0.13499, 0.08897; 2.91081, 2.53524	0.33776
5/3/*h*26	$P\bar{3}c1$ 12*g*	2		0.39875, 0.12304, 0.09712; 2.36368, 2.82537	0.45962
5/3/*h*27	$R\bar{3}c$ 36*f*	1		0.26858, 0.08329, 0.14419; 6.58165, 1.48243	0.33894
5/3/*h*28	$R\bar{3}c$ 36*f*	1		0.26106, 0.08945, 0.24282; 6.45221, 1.69589	0.30829
5/3/*h*29	$R\bar{3}c$ 36*f*	1		0.18851, 0.00256, 0.35200; 3.08331, 4.90148	0.46710
5/3/*h*30	$R\bar{3}c$ 36*f*	2		0.25999, 0.07918, 0.30610; 2.50125, 8.37129	0.41559
5/4/*h*1	$R\bar{3}m$ 18*h*	1	2	0.11180, 0.22360, 0.44375; 5.13140, 1.00000	0.41330
5/4/*h*2	$R\bar{3}m$ 18*h*	1	2	0.11388, 0.22776, 0.11679; 4.92946, 1.00000	0.44786
5/4/*h*3	$R\bar{3}m$ 18*h*	1	2	0.10084, 0.20168, 0.25000; 4.27623, 1.32992	0.44750

Table 3 (continued)

Type	Symmetry	df_{min}	df_{max}	$x, y, z; a, c$	ρ_{min}
5/4/*h*4	$R\bar{3}m$ 18*h*	1	2	0.15403, 0.30806, 0.11849; 3.13953, 2.30522	0.47896
5/4/*h*5	*P*6/*mmm* 2*c*	0	2	0.33333, 0.66667, 0.00000; 1.73205, 1.00000	0.40307
5/4/*h*6	$P6_122$ 12*c*	1		0.45918, 0.04082, 0.37330; 1.87919, 3.94631	>0.51013
	$P3_221$ 6*c*	2		0.44592, 0.01460, 0.42000; 1.84075, 2.02483	
5/4/*h*7	$P6_122$ 12*c*	2		0.36278, 0.06658, 0.34988; 1.78324, 4.81569	0.47377
5/4/*h*8	$P3_221$ 6*c*	1		0.38825, 0.07869, 0.41667; 1.76480, 1.94128	>0.58843
5/4/*h*9	$P3_221$ 6*c*	2		0.50383, 0.06815, 0.41816; 1.88583, 1.96156	0.52001
5/4/*h*10	$P3_221$ 6*c*	1		0.41577, 0.16945, 0.41667; 2.27035, 1.49128	0.47192
5/4/*h*11	$P6_222$ 6*i*	0	2	0.16667, 0.33333, 0.00000; 1.73205, 2.59808	0.46542
5/4/*h*12	$P3_221$ 6*c*	2		0.50000, 0.16667, 0.42812; 1.49723, 3.49730	0.46271
5/4/*h*13	$P6_222$ 6*i*	0	2	0.42020, 0.84041, 0.00000; 3.61767, 1.00000	0.27718
5/4/*h*14	$R\bar{3}m$ 18*f*	0	2	0.42020, 0.00000, 0.00000; 6.26599, 1.00000	0.27718
5/4/*h*15	$P6_222$ 6*g*	0	1	0.32673, 0.00000, 0.00000; 2.88562, 1.00000	0.43565
5/4/*h*16	$P6_222$ 12*k*	1		0.45469, 0.12378, 0.35630; 4.46736, 1.00000	0.36354
5/4/*h*17	*P*6/*mmm* 12*p*	0	1	0.45534, 0.12201, 0.00000; 4.73205, 1.00000	0.32400
5/4/*h*18	*P*6/*mcc* 12*l*	1	1	0.41800, 0.11200, 0.00000; 2.96420, 1.63625	>0.45821
5/4/*h*19	*P*6/*mcc* 12*l*	1	2	0.47385, 0.14051, 0.00000; 3.22060, 1.56762	0.44621
5/4/*h*20	*P*6/*mcc* 12*l*	1	2	0.40396, 0.11227, 0.00000; 2.76885, 1.73573	0.54522
5/4/*h*21	$P6_3/mcm$ 12*j*	1	2	0.48100, 0.14767, 0.25000; 3.90982, 1.37555	0.34503
5/4/*h*22	$P6_222$ 12*k*	0		0.33333, 0.16667, 0.25598; 1.73205, 5.59808	0.43201
5/4/*h*23	$P6_222$ 12*k*	0		0.34341, 0.09202, 0.33333; 2.29663, 2.79181	0.49270
5/4/*h*24	$P6_222$ 12*k*	0		0.40819, 0.15737, 0.35189; 2.29076, 2.63689	0.52432
5/4/*h*25	$P6_222$ 12*k*	1		0.45518, 0.12245, 0.07079; 4.66742, 1.00000	0.33304
5/4/*h*26	$P6_222$ 12*k*	1		0.46116, 0.10610, 0.25000; 3.90722, 1.39195	0.34142
5/4/*h*27	$P6_122$ 12*c*	1		0.45926, 0.12213, 0.08941; 4.65100, 1.00000	0.33539
5/4/*h*28	$P6_122$ 12*c*	1		0.46572, 0.12606, 0.47752; 4.57547, 1.00000	0.34656
5/4/*h*29	$P6_122$ 12*c*	1		0.44496, 0.11729, 0.27438; 4.39274, 1.00000	0.37599
5/4/*h*30	$P6_122$ 12*c*	1		0.36114, 0.07289, 0.27451; 2.83752, 2.07022	0.43527
5/4/*h*31	$P6_122$ 12*c*	1		0.40813, 0.17122, 0.33333; 2.66110, 1.96921	>0.51750
5/4/*h*32	$P6_122$ 12*c*	1		0.42326, 0.19897, 0.35253; 2.54333, 2.16183	>0.50257
5/4/*h*33	$P6_122$ 12*c*	1		0.42850, 0.15033, 0.37010; 2.27305, 3.10259	0.45259
5/4/*h*34	$P6_122$ 12*c*	1		0.43703, 0.06297, 0.37158; 1.87590, 3.84559	>0.53605
5/4/*h*35	$P6_122$ 12*c*	1		0.33333, 0.16667, 0.29739; 1.49723, 6.99460	0.46271
5/4/*h*36	$P6_322$ 12*i*	1		0.45274, 0.11941, 0.15653; 4.59210, 1.00000	0.34405
5/4/*h*37	*P*6/*mcc* 24*m*	1		0.41535, 0.12681, 0.13167; 2.71236, 3.79731	>0.50321
5/4/*h*38	$R\bar{3}$ 18*f*	2		0.26615, 0.07325, 0.38239; 3.80954, 1.78895	0.41918
5/4/*h*39	$R\bar{3}$ 18*f*	2		0.28147, 0.03476, 0.39683; 2.54715, 3.56663	0.47030
5/4/*h*40	$R\bar{3}m$ 36*i*	1		0.26408, 0.07055, 0.05625; 8.13140, 1.00000	0.32918
5/4/*h*41	$R\bar{3}m$ 36*i*	1		0.26770, 0.07080, 0.38321; 7.92946, 1.00000	0.34616
5/4/*h*42	$R\bar{3}c$ 36*f*	1		0.27207, 0.08298, 0.16667; 6.76751, 1.39195	0.34142
5/4/*h*43	$R\bar{3}c$ 36*f*	1		0.24934, 0.06681, 0.00000; 6.72140, 1.25702	0.38327
5/4/*h*44	$R\bar{3}c$ 36*f*	1		0.22055, 0.03249, 0.00000; 4.84886, 1.92408	0.48114
5/4/*h*45	$R\bar{3}c$ 36*f*	1		0.23788, 0.06685, 0.06290; 4.50857, 2.27922	0.46979
5/4/*h*46	$R\bar{3}c$ 36*f*	1		0.23010, 0.02373, 0.04332; 4.46419, 2.37808	0.45926
5/4/*h*47	$R\bar{3}c$ 36*f*	1		0.23281, 0.04525, 0.08981; 3.95753, 2.96727	0.46834
5/4/*h*48	$R\bar{3}c$ 36*f*	1		0.27656, 0.08834, 0.04758; 3.84918, 3.53516	0.41555
6/3/*h*1	$R\bar{3}m$ 18*h*	0	1	0.11581, 0.23162, 0.08333; 4.91565, 1.00000	0.45038
6/3/*h*2	$R\bar{3}m$ 18*h*	1	2	0.13263, 0.26526, 0.10444; 2.51327, 3.90894	0.44076
6/3/*h*3	$P6_222$ 6*i*	0	1	0.21132, 0.42265, 0.00000; 1.93185, 2.12132	0.45821
6/3/*h*4	$P6_122$ 12*c*	1		0.38027, 0.11973, 0.36661; 1.74851, 4.84954	0.48934
6/3/*h*5	$P6_122$ 12*c*	1		0.33333, 0.06833, 0.32081; 1.71781, 5.11079	0.48107
6/3/*h*6	$P6_122$ 12*c*	1		0.33333, 0.06833, 0.34585; 1.71781, 5.11079	0.48107
6/3/*h*7	$P6_122$ 12*c*	1		0.37085, 0.03323, 0.34261; 1.81903, 4.57755	0.47900
6/3/*h*8	$P3_221$ 6*c*	1		0.41973, 0.10081, 0.43589; 1.74582, 2.06344	0.57681
6/3/*h*9	$P3_221$ 6*c*	1		0.50386, 0.06772, 0.41667; 1.89124, 1.95017	0.52006
6/3/*h*10	$P6_122$ 6*b*	1	2	0.42154, 0.84307, 0.25000; 1.72871, 2.66774	0.45502
6/3/*h*11	$P3_221$ 6*c*	1		0.55196, 0.21862, 0.03138; 1.70466, 2.82269	0.44226
6/3/*h*12	$P6_122$ 6*b*	0	1	0.42616, 0.85232, 0.25000; 3.38562, 1.00000	0.31648
6/3/*h*13	*P*6/*mmm* 3*f*	0	1	0.50000, 0.00000, 0.00000; 2.00000, 1.00000	0.45345
6/3/*h*14	*R*3*m* 9*b*	1	1	0.13333, 0.26667, 0.00000; 2.50000, 1.93649	0.44959
6/3/*h*15	$R\bar{3}c$ 18*e*	0	1	0.42616, 0.00000, 0.25000; 5.86406, 1.00000	0.31648
6/3/*h*16	$R\bar{3}c$ 18*e*	1	2	0.42154, 0.00000, 0.25000; 2.99422, 2.66774	0.45502
6/3/*h*17	$P6_122$ 6*a*	0	1	0.34743, 0.00000, 0.00000; 2.83805, 1.00000	0.45038
6/3/*h*18	*R*32 9*d*	1	1	0.43670, 0.00000, 0.00000; 1.95359, 2.81051	0.50729

Table 3 (continued)

Type	Symmetry	df_{min}	df_{max}	$x, y, z; a, c$	ρ_{min}
6/3/*h*19	$R\bar{3}c$ 18*e*	0	1	0.19098, 0.00000, 0.25000; 3.02305, 2.00000	0.59542
	$R\bar{3}$ 18*f*	1		0.20389, 0.01672, 0.29829; 2.94478, 2.12233	0.59132
6/3/*h*20	*P*6/*mmm* 6*l*	0	2	0.21132, 0.42265, 0.00000; 2.73205, 1.00000	0.48601
6/3/*h*21	$P6_3/mmc$ 6*h*	0	1	0.42857, 0.85714, 0.25000; 3.50000, 1.00000	0.29613
6/3/*h*22	$P6_3/mmc$ 6*h*	1	2	0.18567, 0.37133, 0.25000; 2.25733, 1.37555	0.51755
6/3/*h*23	*R*3*c* 18*b*	1		0.19810, 0.02599, 0.00000; 3.09629, 1.78361	0.63644
6/3/*h*24	*R*3*c* 18*b*	1		0.31333, 0.12281, 0.00000; 3.27572, 1.95017	0.52006
6/3/*h*25	*R*3*c* 18*b*	1		0.21983, 0.07254, 0.00000; 2.97590, 1.94989	0.63022
6/3/*h*26	$P6_1$ 6*a*	1		0.46984, 0.10615, 0.00000; 2.25175, 1.65903	0.43125
6/3/*h*27	$P\bar{6}2c$ 6*h*	1	1	0.33333, 0.10337, 0.25000; 1.95359, 1.87367	0.50729
6/3/*h*28	$P6_3/mmc$ 12*j*	0	1	0.44444, 0.11111, 0.25000; 4.50000, 1.00000	0.35828
6/3/*h*29	$P6_222$ 12*k*	0		0.46144, 0.10534, 0.25000; 3.87550, 1.41421	0.34157
6/3/*h*30	$P6_222$ 12*k*	0		0.45292, 0.12862, 0.41667; 4.42589, 1.00000	0.37038
6/3/*h*31	$P6_3/m$ 12*i*	1		0.41442, 0.12565, 0.07291; 2.47601, 2.82349	0.41914
6/3/*h*32	$P6_122$ 12*c*	0		0.46842, 0.12633, 0.00000; 4.57010, 1.00000	0.34737
6/3/*h*33	$P6_122$ 12*c*	0		0.44300, 0.11401, 0.25000; 4.38562, 1.00000	0.37721
6/3/*h*34	$P6_122$ 12*c*	0		0.44924, 0.12510, 0.33333; 4.35106, 1.00000	0.38323
6/3/*h*35	$P6_122$ 12*c*	0		0.34732, 0.04197, 0.25000; 2.87915, 1.95570	0.44753
6/3/*h*36	$P6_122$ 12*c*	0		0.40521, 0.16352, 0.33333; 2.67744, 1.95570	0.51750
6/3/*h*37	$P6_122$ 12*c*	0		0.42523, 0.18415, 0.36939; 2.49806, 2.31337	0.50257
6/3/*h*38	$P6_122$ 12*c*	0		0.43726, 0.06274, 0.37215; 1.88024, 3.82838	0.53605
6/3/*h*39	$P6_322$ 12*i*	1		0.33333, 0.13512, 0.11788; 1.98816, 3.75452	0.48887
6/3/*h*40	$P\bar{6}2c$ 12*i*	1		0.33333, 0.12724, 0.11839; 1.98160, 3.79920	0.48632
6/3/*h*41	*P*6/*mcc* 24*m*	0		0.48316, 0.14983, 0.13763; 3.14626, 3.63299	0.40348
6/3/*h*42	*P*6/*mcc* 24*m*	0		0.40013, 0.10722, 0.13473; 2.78745, 3.71120	0.50321
6/3/*h*43	$P6_3/mcm$ 24*l*	0		0.50000, 0.16667, 0.11237; 3.46410, 3.63299	0.33284
6/3/*h*44	$R\bar{3}$ 18*f*	1		0.25668, 0.04340, 0.27457; 2.42621, 3.94920	0.46814
6/3/*h*45	$R\bar{3}$ 18*f*	1		0.27784, 0.08585, 0.41667; 3.90014, 1.65903	0.43125
6/3/*h*46	$R\bar{3}$ 18*f*	2		0.19670, 0.01312, 0.29279; 3.03106, 1.97019	>0.59198
6/3/*h*47	$R\bar{3}$ 18*f*	1		0.30229, 0.02415, 0.28372; 2.81399, 2.91699	0.47115
6/3/*h*48	*R*32 18*f*	1		0.33333, 0.12724, 0.07893; 1.98160, 5.69881	0.48632
6/3/*h*49	$P\bar{3}1c$ 12*i*	1		0.36389, 0.05442, 0.07052; 2.83072, 1.92750	0.46975
6/3/*h*50	$P\bar{3}1c$ 12*i*	1		0.41447, 0.12572, 0.07751; 2.47660, 2.65185	0.44605
6/3/*h*51	$P\bar{3}c1$ 12*g*	1		0.46635, 0.13302, 0.03207; 3.25996, 1.51470	0.45071
6/3/*h*52	$P\bar{3}c1$ 12*g*	1		0.45887, 0.14485, 0.05336; 2.90349, 1.74209	0.49401
6/3/*h*53	$P\bar{3}c1$ 12*g*	1		0.40246, 0.11030, 0.00446; 2.77583, 1.72644	0.54539
6/3/*h*54	$P\bar{3}c1$ 12*g*	1		0.40429, 0.11270, 0.09234; 2.36433, 2.81346	0.46131
6/3/*h*55	$R\bar{3}m$ 36*i*	0		0.27017, 0.07192, 0.41667; 7.91565, 1.00000	0.34737
6/3/*h*56	$R\bar{3}m$ 36*i*	0		0.25948, 0.06487, 0.25000; 7.70783, 1.00000	0.36636
6/3/*h*57	$R\bar{3}c$ 36*f*	1		0.26048, 0.07422, 0.30625; 7.72948, 1.00000	0.36431
6/3/*h*58	$R\bar{3}c$ 36*f*	1		0.26420, 0.07458, 0.13321; 7.52754, 1.00000	0.38412
6/3/*h*59	$R\bar{3}c$ 36*f*	0		0.27251, 0.08359, 0.16667; 6.71257, 1.41421	0.34157
6/3/*h*60	$R\bar{3}c$ 36*f*	1		0.22773, 0.07281, 0.03570; 4.92150, 1.82941	0.49121
6/3/*h*61	$R\bar{3}c$ 36*f*	1		0.21852, 0.08376, 0.42707; 5.02440, 1.93322	0.44599
6/3/*h*62	$R\bar{3}c$ 36*f*	1		0.22053, 0.09412, 0.09451; 4.84010, 1.97543	0.47033
6/3/*h*63	$R\bar{3}c$ 36*f*	1		0.18301, 0.07709, 0.12069; 3.62789, 3.38270	0.48887
6/3/*h*64	$R\bar{3}c$ 36*f*	1		0.17866, 0.05300, 0.38397; 3.63264, 3.51849	0.46878
6/3/*h*65	$R\bar{3}c$ 36*f*	1		0.21061, 0.06402, 0.08914; 3.08730, 4.58006	0.49859
6/3/*h*66	$R\bar{3}c$ 36*f*	1		0.24205, 0.07971, 0.33441; 2.70223, 5.49581	0.54237
6/3/*h*67	$R\bar{3}c$ 36*f*	1		0.25305, 0.04436, 0.30828; 2.46683, 8.42353	0.42462
6/3/*h*68	$R\bar{3}c$ 36*f*	1		0.25638, 0.04348, 0.36063; 2.42947, 8.44244	0.43680
6/3/*h*69	$R\bar{3}c$ 36*f*	1		0.33333, 0.13512, 0.21071; 1.98816, 11.26357	0.48887
6/4/*h*1	$R\bar{3}m$ 9*e*	0	1	0.50000, 0.00000. 0.00000; 3.26599, 1.00000	0.51013
6/4/*h*2	$P6_3/mmc$ 2*c*	1	2	0.33333, 0.66667. 0.25000; 1.41421, 1.15470	0.52360
6/4/*h*3	$P6_222$ 3*c*	0	1	0.50000, 0.00000, 0.00000; 1.88562, 1.00000	0.51013
6/4/*h*4	$P3_221$ 6*c*	1		0.35584, 0.02251, 0.42658; 1.64943, 2.07828	>0.58365
6/4/*h*5	$P6_222$ 6*i*	0	1	0.21132, 0.42265, 0.00000; 2.57580, 1.00000	0.54676
6/4/*h*6	$P6_122$ 6*b*	0	1	0.21477, 0.42954, 0.25000; 2.65063, 1.00000	0.51632
6/4/*h*7	*R*32 9*d*	0	1	0.17157, 0.00000, 0.00000; 3.36504, 1.00000	0.48054
6/4/*h*8	$R\bar{3}c$ 18*e*	1	2	0.19854, 0.00000, 0.25000; 3.87549, 1.27745	0.56721
6/4/*h*9	$R\bar{3}m$ 18*f*	0	2	0.21584, 0.00000, 0.00000; 4.63299, 1.00000	0.50701
6/4/*h*10	*P*6/*mcc* 12*l*	1	1	0.45534, 0.12201, 0.00000; 3.86370, 1.15470	0.42089
6/4/*h*11	$R\bar{3}c$ 36*f*	0		0.24634, 0.05146, 0.06875; 4.15827, 2.56221	0.49128

Table 3 (continued)

Type	Symmetry	df_{min}	df_{max}	$x, y, z; a, c$	ρ_{min}
7/3/*h*1	$R\bar{3}m$ 18*h*	0	1	0.10621, 0.21241, 0.25000; 4.70783, 1.00000	0.49102
7/3/*h*2	$R\bar{3}m$ 18*h*	0	1	0.09091, 0.18182, 0.25000; 3.66667, 1.63299	0.49569
7/3/*h*3	$R\bar{3}m$ 18*h*	0	1	0.12209, 0.24417, 0.15794; 2.73029, 2.58490	0.56478
7/3/*h*4	$R\bar{3}m$ 18*h*	0	1	0.16667, 0.33333, 0.24158; 2.00000, 5.44949	0.49926
7/3/*h*5	$R\bar{3}c$ 36*f*	0		0.23139, 0.08184, 0.05648; 4.80693, 1.89109	0.49811
7/3/*h*6	$P6_122$ 12*c*	0		0.43670, 0.06330, 0.37079; 1.86969, 3.86969	0.53633
7/3/*h*7	$P6_122$ 12*c*	0		0.33333, 0.16667, 0.30381; 1.64575, 5.27963	0.50736
7/3/*h*8	$P6_122$ 12*c*	0		0.33333, 0.00000, 0.33333; 1.73205, 4.89898	0.49365
7/3/*h*9	$P3_221$ 6*c*	0		0.40515, 0.10129, 0.41667; 1.80255, 1.89737	0.58843
7/3/*h*10	$P3_221$ 6*c*	1		0.50963, 0.06871, 0.42329; 1.86455, 2.00321	0.52089
7/3/*h*11	$P3_221$ 6*c*	1		0.55664, 0.22331, 0.43411; 1.60469, 2.85629	0.49321
7/3/*h*12	$P6_2$ 6*c*	0		0.42703, 0.15203, 0.00000; 2.51484, 1.00000	0.57359
7/3/*h*13	$P6/m$ 6*j*	0	1	0.42857, 0.14286, 0.00000; 2.64575, 1.00000	0.51823
7/3/*h*14	$P6/mcc$ 12*l*	0	0	0.45534, 0.12201, 0.00000; 3.34607, 1.41421	0.45821
7/3/*h*15	$P6/mcc$ 12*l*	0	1	0.48316, 0.14983, 0.00000; 3.14626, 1.63299	0.44882
7/3/*h*16	$P6/mcc$ 12*l*	0	0	0.40013, 0.10722, 0.00000; 2.78745, 1.71120	0.54567
7/3/*h*17	$P6/mcc$ 12*l*	0	1	0.42857, 0.14286, 0.00000; 2.64575, 1.85164	0.55975
7/3/*h*18	$P6_3/mcm$ 12*j*	0	1	0.46667, 0.13333, 0.25000; 4.33013, 1.00000	0.38694
7/3/*h*19	$P6_3/mcm$ 12*j*	0	1	0.50000, 0.16667, 0.25000; 3.46410, 1.63299	0.37024
7/3/*h*20	$P6_3/mmc$ 12*k*	0	1	0.16667, 0.33333, 0.11237; 2.00000, 3.63299	0.49926
7/3/*h*21	$P6_222$ 12*k*	0		0.45781, 0.11526, 0.25000; 4.33805, 1.00000	0.38553
7/3/*h*22	$P6/mcc$ 24*m*	0		0.42857, 0.14286, 0.12981; 2.64575, 3.85164	0.53819
7/3/*h*23	$R\bar{3}$ 18*f*	1		0.20519, 0.01855, 0.30844; 2.93706, 2.13113	0.59198
7/3/*h*24	$R\bar{3}$ 18*f*	1		0.20816, 0.02319, 0.30955; 2.92199, 2.14357	0.59463
7/3/*h*25	$R\bar{3}$ 18*f*	1		0.24275, 0.04757, 0.03466; 4.47752, 1.00000	0.54283
7/3/*h*26	$R\bar{3}$ 18*f*	1		0.21327, 0.02995, 0.25000; 3.77660, 1.31086	0.58208
7/3/*h*27	$R\bar{3}$ 18*f*	1		0.31237, 0.11328, 0.38440; 3.14556, 2.19553	0.50096
7/3/*h*28	$R\bar{3}$ 18*f*	1		0.25494, 0.04387, 0.39211; 2.44555, 3.78376	0.48091
7/3/*h*29	$R\bar{3}$ 18*f*	1		0.29960, 0.02573, 0.39421; 2.74062, 2.90845	0.49817
7/3/*h*30	$P3_221$ 6*c*	0		0.42018, 0.19648, 0.41667; 2.54481, 1.00000	0.56016
7/3/*h*31	$P\bar{3}c1$ 12*g*	1		0.41570, 0.16735, 0.08285; 2.61148, 1.95474	0.54424
7/3/*h*32	$P\bar{3}c1$ 12*g*	1		0.33333, 0.12429, 0.11859; 1.97879, 3.44245	0.53825
7/3/*h*33	$R\bar{3}c$ 36*f*	0		0.26679, 0.07576, 0.16667; 7.51373, 1.00000	0.38553
7/3/*h*34	$R\bar{3}c$ 36*f*	0		0.25541, 0.06844, 0.00000; 7.30590, 1.00000	0.40778
7/3/*h*35	$R\bar{3}c$ 36*f*	0		0.22280, 0.05970, 0.00000; 5.00618, 1.71120	0.50752
7/3/*h*36	$R\bar{3}c$ 36*f*	0		0.18926, 0.02851, 0.11319; 3.26676, 3.60680	0.56547
7/3/*h*37	$R\bar{3}c$ 36*f*	0		0.23820, 0.04760, 0.33670; 2.64439, 5.62809	0.55304
7/3/*h*38	$R\bar{3}c$ 36*f*	1		0.33333, 0.12429, 0.03953; 1.97879, 10.32735	0.53825
7/4/*h*1	$R\bar{3}$ 18*f*	1		0.24841, 0.05780, 0.41537; 4.37753, 1.00000	0.56791
8/3/*h*1	$R\bar{3}$ 18*f*	0		0.24857, 0.05798, 0.41667; 4.37751, 1.00000	0.56792
8/3/*h*2	$R\bar{3}c$ 18*e*	0	0	0.46785, 0.00000, 0.25000; 3.25479, 1.95570	0.52528
8/3/*h*3	$P6_3/mmc$ 2*c*	0	1	0.33333, 0.66667, 0.25000; 1.50000, 1.00000	0.53742
8/3/*h*4	$P6/mmm$ 1*a*	0	2	0.00000, 0.00000, 0.00000; 1.00000, 1.00000	0.60460
8/3/*h*5	$P3_221$ 6*c*	0		0.43218, 0.09885, 0.44769; 1.68614, 2.18614	0.58365
8/3/*h*6	$P6_122$ 6*b*	0	0	0.46785, 0.93569, 0.25000; 1.87915, 1.95570	0.52528
8/3/*h*7	$R3m$ 9*b*	0	0	0.11556, 0.23113, 0.00000; 2.88444, 1.00000	0.65402
8/3/*h*8	$R\bar{3}c$ 18*e*	0	1	0.20468, 0.00000, 0.25000; 4.23107, 1.00000	0.60791
8/3/*h*9	$R\bar{3}c$ 18*e*	0	1	0.18127, 0.00000, 0.25000; 3.18502, 1.63299	0.65695
8/3/*h*10	$P6_3/mmc$ 6*h*	0	1	0.16667, 0.33333, 0.25000; 2.00000, 1.63299	0.55536
8/3/*h*11	$P6_3/mmc$ 6*h*	0	1	0.20000, 0.40000, 0.25000; 2.50000, 1.00000	0.58042
8/3/*h*12	$P\bar{3}c1$ 12*g*	0		0.48051, 0.14717, 0.06250; 3.10136, 1.63299	0.46191
8/3/*h*13	$P6_1$ 6*a*	0		0.43915, 0.13260, 0.00000; 2.52736, 1.00000	0.56792
8/3/*h*14	$P6_3/m$ 6*h*	1	1	0.38762, 0.09006, 0.25000; 2.19531, 1.27288	0.59135
8/3/*h*15	$P6/mcc$ 12*l*	0	0	0.45534, 0.12201, 0.00000; 4.09808, 1.00000	0.43201
8/3/*h*16	$P\bar{3}c1$ 12*g*	0		0.42674, 0.14068, 0.04201; 2.62278, 1.84892	0.57044
9/3/*h*1	$R\bar{3}$ 18*f*	0		0.22574, 0.03884, 0.25000; 4.14278, 1.00000	0.63410
9/3/*h*2	$R\bar{3}$ 18*f*	0		0.18681, 0.01099, 0.25000; 3.17980, 1.63299	0.65911
9/3/*h*3	$P6_222$ 6*f*	0	2	0.50000, 0.00000, 0.08932; 1.00000, 5.59808	0.64801
10/3/*h*1	$R\bar{3}$ 18*f*	0		0.23810, 0.04762, 0.33333; 2.64575, 2.44949	0.63470
10/3/*h*2	$P6_3/mmc$ 4*f*	0	1	0.33333, 0.66667, 0.11237; 1.00000, 3.63299	0.66568
10/3/*h*3	$P6_222$ 3*c*	0	2	0.50000, 0.00000, 0.00000; 1.00000, 2.59808	0.69813
10/3/*h*4	$R\bar{3}m$ 6*c*	0	1	0.00000, 0.00000, 0.09175; 1.00000, 5.44949	0.66568
10/3/*h*5	$P6_3/m$ 6*h*	0	0	0.40644, 0.11549, 0.25000; 2.38736, 1.00000	0.63648

Table 3 (continued)

Type	Symmetry	df_{min}	df_{max}	$x, y, z; a, c$	ρ_{min}
11/3/*h*1	$P6_122$ 12*c*	0		0.33333, 0.16667, 0.29289; 1.00000, 10.09513	0.71868
11/3/*h*2	$P3_221$ 6*c*	1		0.50000, 0.16667, 0.41912; 1.00000, 5.04757	0.71868
12/3/*h*1	$P6_3/mmc$ 2*c*	0	0	0.33333, 0.66667, 0.25000; 1.00000, 1.63299	0.74048

consideration but are located only on their boundaries. Therefore, it is impossible to deform a sphere packing (interpenetrating sphere packings) from the first parameter range into another packing from the second range without allowing additional contacts between spheres during the deformation process. As a consequence, a packing with a higher number of contacts per sphere must be formed in the course of the deformation. For 4/6/*h*14 (*cf.* Fig. 2), 5/4/*h*6 and $h[3/10/h1]^2$, it belongs to type 6/4/*h*3. Similar cases have been discussed before for some tetragonal types of sphere packing (*cf.* Koch & Sowa, 2004; Fischer, 2005) and some types of interpenetrating sphere packings (Koch *et al.*, 2006).

Type 6/3/*h*19 shows another exceptional property that – until now – has only been observed once. It occurs with no degree of freedom and with site symmetry .2 in $R\bar{3}c$ 18*e* and with one degree of freedom and site symmetry 1 in *R*3*c* 18*b* as well as in $R\bar{3}$ 18*f*. The common limiting complex of *R*3*c* 18*b* and $R\bar{3}$ 18*f* is $R\bar{3}c$ 18*e*. The density of a sphere packing with symmetry $R\bar{3}c$ 18*e* is $\rho = 0.59542$ and agrees, as usual, with the minimal density of a sphere packing with symmetry *R*3*c* 18*b*. In contrast to that, the sphere packings with minimal density and symmetry $R\bar{3}$ 18*f* do not belong to the limiting complex, so that the respective value of the minimal density is slightly

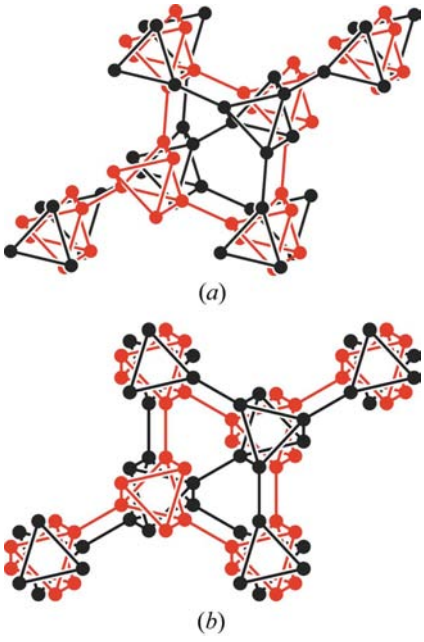

Figure 1
Two sphere packings of type 4/3/*h*5 forming interpenetrating sphere packings of types: (*a*) $h[4/3/h5]^2_{\rm I}$ with symmetry *R*32, (*b*) $h[4/3/h5]^2_{\rm II}$ with symmetry $R\bar{3}c$.

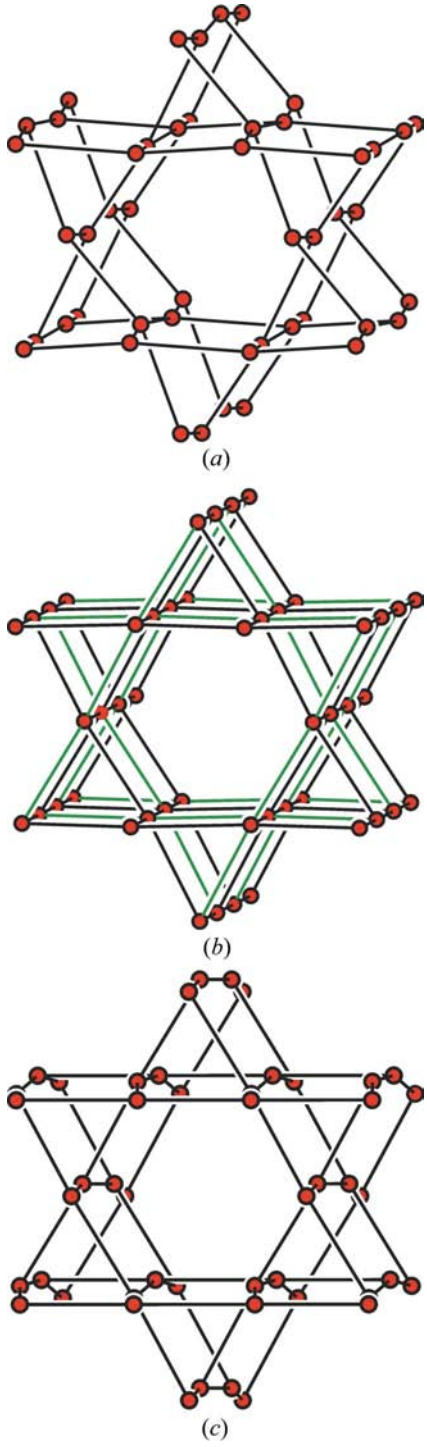

Figure 2
Sphere packings of type 4/6/*h*14 with symmetries (*a*) $P6_122$ and (*c*) $P3_221$. (*b*) Type 6/4/*h*3 refers to the common limiting complex $P6_222$ 3*c* of $P6_122$ 12*c* and $P3_221$ 6*c*. The green lines show the additional sphere contacts.

lower, namely $\rho = 0.59132$. Sphere-packing densities have been discussed in more detail in a previous paper (Koch *et al.*, 2005).

The lists of trigonal and hexagonal sphere packings given in the present and in some previous papers (Sowa *et al.*, 2003; Sowa & Koch, 2004, 2005) are assumed to be complete because for each lattice complex an entire asymmetric unit of the Euclidean normalizer of its characteristic space group has been investigated. This is also true for the lists of sphere packings with cubic and tetragonal symmetry published by Fischer (2004, 2005). Compared with this, combinatorial methods seem to involve a certain risk to produce incomplete or incorrect results. In the tabulation of types of sphere packing with three contacts per sphere by Koch & Fischer (1995), six cases are missing. The enumeration of 'periodic tetrahedral frameworks' by Treacy *et al.* (1997) comprises (among others) 44 frameworks that correspond to 44 of the 74 types of sphere packing with contact number four and with hexagonal or trigonal symmetry. In some of the 'tetrahedral frameworks', however, the number of nearest neighbours is five (Nos. 70, 82, 93, 132) or six (No. 102) instead of four and, as a consequence, the frameworks with Nos. 70 and 82 and with Nos. 93 and 132 are the same. In other frameworks, a fifth neighbour exists with a distance that is much shorter than the other four (Nos. 60, 64).

Table 3 contains a compilation of all 317 types of sphere packing that occur with maximal symmetry in the hexagonal crystal family. It is analogous to previously published tables on sphere-packing types with cubic and with tetragonal symmetry (Fischer, 2004, 2005). In the first column, each type is identified by its symbol. Column 2 gives the maximal symmetry of a sphere packing of that type. The different parameter regions that belong to a certain type of sphere packing may differ in their dimensions, *i.e.* a sphere-packing type may occur with different numbers *df* of degrees of freedom. The table shows the minimal and maximal values of *df* that occur within the hexagonal crystal family. If a type comprehends a sphere packing with minimal density ρ_m, the corresponding value is tabulated together with the coordinate and lattice parameters in the last two columns. If such a minimal density does not exist, the lower limit of ρ is given together with parameters for an arbitrarily chosen sphere packing of that type. All lattice parameters are calculated for a distance $d = 1$ between neighbouring spheres.

4. Structural examples

In the following, some examples are given describing atomic arrangements that are very close to sphere packings. All regarded crystal structures contain atomic positions that refer to one of the trigonal lattice complexes with three degrees of freedom investigated in the present paper. The examples are arbitrarily chosen and the authors do not lay claim to completeness.

(i) B_2O_3 crystallizes with space group $P3_121$ (axial ratio c/a = 1.9235; *cf.* Effenberger *et al.*, 2001). The arrangement of the B atoms ($x = 0.395$, $y = 0.230$, $z = 0.244$) corresponds approximately to a sphere packing of type 3/10/*h*1 (B_2O_3 net by O'Keeffe & Hyde, 1996).

(ii) The zeolite chabazite SiO_2 (CHA) crystallizes with space group $R\bar{3}m$. The arrangement of the Si atoms corresponds to a sphere packing of type 4/4/*h*7. The structural parameters [$x = 0.22862$, $y = 0.00002$, $z = 0.10389$, $c/a = 1.0901$, *cf.* Diaz-Cabañas *et al.* (1998)] are very close to the parameters of the sphere packing with minimal density ($x = 0.22907$, $y = 0$, $z = 0.10319$, $c/a = 1.10993$).

(iii) $R\bar{3}$ is the space group of the zeolite ATO ($AlPO_4$-31) ($c/a = 0.2402$; Bennett & Kirchner, 1992). The P atoms (at x = 0.246, $y = 0.052$, $z = 0.289$) and the Al atoms (at $x = 0.247$, y = 0.199, $z = 0.198$) form together an arrangement that is similar to a sphere packing with minimal density belonging to type 4/4/*h*10 in the supergroup $R\bar{3}m$ of $R\bar{3}$ ($c/a = 0.2263$; $x = 0.252$, $y = 0.053$, $z = \frac{1}{4}$).

(iv) The O atoms in α-quartz ($P3_221$, $c/a = 1.1007$) at x = 0.4151, $y = 0.2675$ and $z = 0.7861$ (Glinnemann *et al.*, 1992) build up a sphere packing of type 6/3/*h*3. The Si atoms in the tetrahedral voids are arranged according to a sphere packing of type 4/6/*h*1.

(v) BaH_3IO_6 crystallizes with space group $R32$ (c/a = 0.9522). The O atoms are located at the general position 18*f* with $x = 0.2262$, $y = 0.0831$, $z = 0.3457$ (Sasaki *et al.*, 1995) and form a sphere packing of type 4/3/*h*5, while the Ba atoms at Wyckoff positions 3*a* 0,0,0 and the I atoms at 3*b* 0,0,$\frac{1}{2}$ are arranged as distorted cubic primitive lattices (type 6/4/*c*1). The Ba atoms are surrounded by 12 O atoms forming a distorted icosahedron whereas the I atoms are octahedrally coordinated by O atoms.

(vi) The symmetry of Ca_3FeRhO_6 is $R\bar{3}c$ with $c/a = 1.1727$ (Davis *et al.*, 2003). The O atoms ($x = 0.1791$, $y = 0.0216$, z = 0.1195) correspond to a slightly distorted sphere packing of type 7/3/*h*36. The Ca atoms are arranged in a sphere packing of type 4/6/*c*2, the Fe atoms as well as the Rh atoms are in a sphere packing of type 8/4/*c*1 (body-centred cubic lattice of spheres). Eight O atoms forming a distorted square antiprism coordinate each Ca atom. The Fe atoms are located in O-atom octahedra, the Rh atoms in slightly distorted trigonal prisms. The octahedra and the prisms alternate in infinite chains of face-sharing polyhedra.

(vii) A large number of structures contain hexagonally closest packed atoms. For instance, the anions in ilmenite- and BiI_3-type structures (*e.g.* Wechsler & Prewitt, 1984; Ruck, 1995) crystallizing in $R\bar{3}$ form slightly distorted sphere packings of type 12/3/*h*1. In both structure types, the cations are octahedrally coordinated. There are several structure types with symmetry $P\bar{3}1c$ where the anions are hexagonally closest packed and the cations occupy octahedral voids in different distributions (*e.g.* Takahashi & Yamada, 1973; Beck *et al.*, 1993; Maletka *et al.*, 1998; Wada & Sato, 1998). The Ti atoms in Ti_6O with symmetry $P\bar{3}1c$ are also arranged similar to a packing of type 12/3/*h*1 (Fykin *et al.*, 1970).

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